

Table of Contents

1. The Nature of Polyphony 1
 - Melodic Structure 1
 - Analysis of Voice Leading 3
2. Writing Simple Melodies 10
3. Introduction to Counterpoint 18
 - Why Study Counterpoint? 18
 - A Brief History of Contrapuntal Theory 18
4. Basic Contrapuntal Technique 21
 - A Historical Note about Consonance and Dissonance 21
 - Species Counterpoint 22
 - Consonance and Dissonance (Harmonic) 22
 - General Principles 22
5. First Species, 1:1, The Foundation 24
 - Thirds and Sixths 24
 - Dissonant Intervals 24
 - Fifths, Octaves, and Unisons 24
 - Examples of First Species 25
 - Chromaticism in First Species 27
6. Florid Counterpoint 28
 - Second Species, 2:1 28
 - 3:1 and 4:1 (Third Species) Counterpoint 33
 - Fourth Species: Syncopation 34
 - Fifth Species, Free Counterpoint 35
7. Form Generating Techniques 36
 - Imitative Counterpoint 36
 - Canons 37
 - How to Write a Canon 38
 - Special Types of Canons 39
 - Rounds, or “Perpetual Canons” 39
 - Canons in Contrary Motion 39
 - Crab Canons 40
 - The Spiral Canon 43

Canons of Augmentation and Diminution 45
Invertible Counterpoint 45

8. Writing Short Contrapuntal Pieces 48
 The Invention 48
 Ground Bass Variations 49
 The Binary Dance Suite Movement 49

9. Fugue 51

10. Dissonant Counterpoint 53

Appendix A. A Schenkerian Analysis of Bach's C Major Prelude, WTC I. 55
 An Algorithm for Graphic Analysis 55
 Theory 55
 The Choice of a Model 56
 The Algorithm: Foreground 57
 The Foreground 59
 Middleground 61
 The Background 65
 A Comparison with Schenker's Analysis 65
 Schenkerian Graphs: Bach's C Major Prelude, WTC I 68

Appendix B. Symbols for Schenkerian Graphs 76

1. The Nature of Polyphony

Polyphony is a texture of music in which there is more than one voice part at a time. Its opposite is monophony, which is a single, unaccompanied melody, as in Gregorian chant or unadorned folksongs. Polyphony is ordinarily divided into two categories: 1. homophony, and 2. counterpoint. Homophony consists of a melody with chordal accompaniment. It is primarily concerned with the vertical dimension. Counterpoint is the weaving together of two or more independent melodies simultaneously, with a harmonic consistency -- a more complex, advanced procedure. Its focus is primarily, although not completely, horizontal.

In *The Structure of Diatonic Music* we were primarily concerned with homophony, i.e., the vertical dimension. Thus, we analyzed and created music by means of “harmonizing” melodies with diatonic chords in a key. In reality, however, counterpoint and homophony cannot be separated, for homophonic music contains counterpoint, and contrapuntal music contains harmony. The Bach chorales, for example, although they are conventionally viewed as homophonic in structure, are also contrapuntal, because the voices consist of independent lines and it is independence that is the primary difference between homophony and counterpoint. Thus, in “harmonizing” a chorale tune we embrace both harmony and counterpoint, although one may not be so aware of the latter. However, the combining of melodic lines comes closer to a contrapuntal ideal when they are maximally different from one another, i.e., rhythmically, directionally, etc.

The Structure of Polyphonic Music focuses on the horizontal dimension of music which has its basis in melody. Even a single line, as we will see, may contain both implied harmony and counterpoint. And, a combination of any two lines without regard for their harmonic content is not true counterpoint but simply a chance juxtaposition.

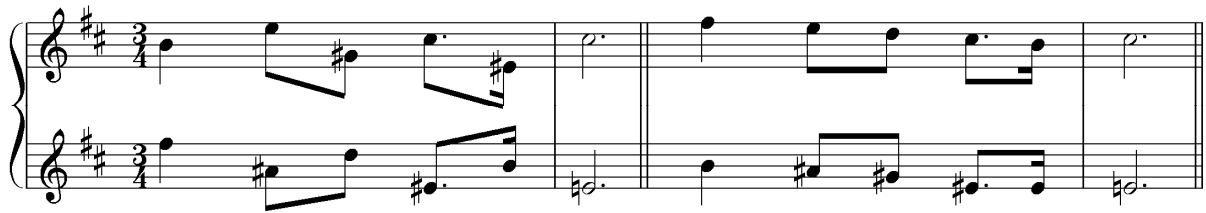
Melodic Structure

Since polyphony is made up of independent melodic lines, we will begin with the structure of a single melodic line. Two fundamental parameters that make up melody are: 1. a series of pitches, and 2. rhythm. Monophonic rhythm, our starting point, consists of two aspects: 1. duration, and 2. accent. Durations are determined by the length of notes, e.g., half, quarters, eighths, etc. Accents are normally fixed by the meter. Thus, we can refer to strong and weak beats and to notes that are on or off the beat. In a compound meter, such as $\mathfrak{6}$, there are two beats of which the first is strong and the second is weak. Likewise, there are six pulses of unequal importance. A note occurring on the second beat is weak with respect to the first beat but strong compared to pulses five and six. (Refer to the *Music Fundamentals* text for a review of meter.)

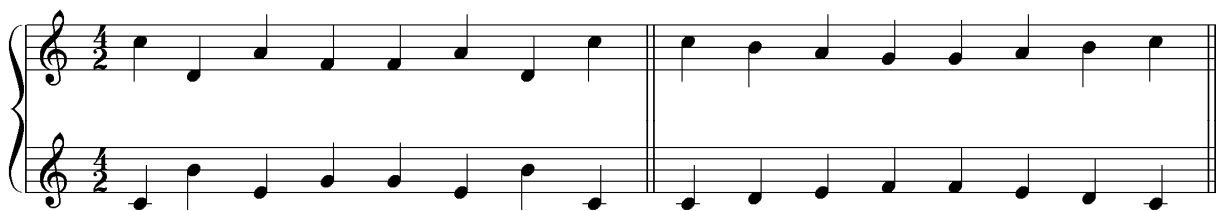
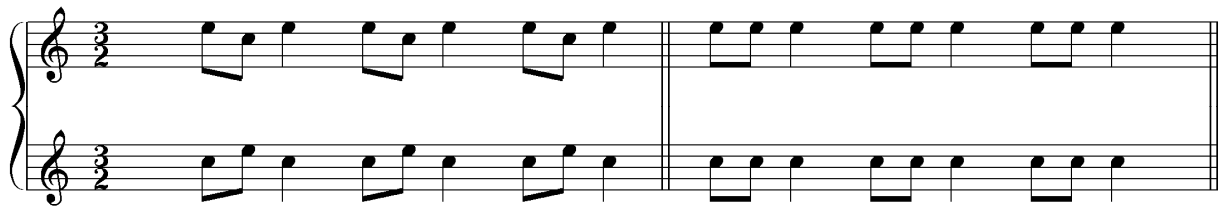
A melody, however, is more than an arbitrary string of notes and rhythmic values. We can say that some melodies are more satisfying than others. Thus, perception and cognition play a part in determining satisfactory and unsatisfactory melodies. For this reason we need to examine how we perceive melodic lines. Recent psychological studies have shown that the mind is continually and involuntarily trying to make sense of the sensory data that it receives from our ears, eyes, nose, etc. This is done by making connections, or relationships, between the incoming data and data that is already stored in memory. Sometimes this is easy and at others it is very difficult. The mind tends to favor the easiest connections which are usually also the shortest routes. In a polyphonic texture, the ear and mind tend to form melodic lines from notes that are close together in pitch, i.e., those that repeat or move stepwise. Leaps are less likely to make sense melodically and are more difficult, or

challenging, to connect. The larger and more frequent the leaps the more difficult and unlikely the melody becomes. So, as a rule, step motion predominates in “good” melodies. Step motion is not only easier to hear and understand, but it is also easier to sing and play.

Ex1a shows the beginning notes of the last movement of Tchaikovsky’s *Symphony No. 6*. The violins play leaping lines with no step motion. But when this is played by two instruments of the same timbre, what is heard is the illusion of a stepwise descending line shown as Ex1b. (It is recommended that students play this.)



This amazing aural illusion proves that the mind prefers smooth motion, or shortest routes, over leaps for purposes of comprehension. Here are some others:



Analysis of Voice Leading

We have learned that the ear and mind tend to connect melodic notes by listening for step motion even when step motion is not being played or is not apparent in the notation. There are many examples, especially in instrumental music, where the principle of smooth motion seems to be violated. The following example is from Bach's Prelude in C major in his *Well-Tempered Clavier* (WTC), Book I.

C: I ii₂⁴ V₅⁶ I

Th

e “lines” here seem to leap from note to note as arpeggiated chords, the first measure being tonic and the second being a II⁷, etc. A reduction of these four measures discloses the essential voice leading (Ex1):

1. preliminary reduction 2. foreground 3. middleground 4. background

C: I ii₂⁴ V₅⁶ I I (domain) I

Five voices are revealed: 2 sopranos, 1 alto, 1 tenor, and 1 bass. Each is moving stepwise as smoothly as possible. In Ex 1 each measure is compressed to a single chord and contains all the notes of the original version. Ex 2 displays the basic structure of the voice leading, called the *foreground* in Schenkerian¹ terminology. It “sees” the first four measures as simply tonic with nonharmonic step motion in and out of the notes of the tonic. All the notes are still represented. On a more “distant” level, or *middleground*, Ex 3 simplifies the first four measures as simply a *prolongation* of tonic. This is accomplished by deleting notes of lesser importance. Finally, Ex 4 shows these four measures to be essentially a tonic which is part of a larger directed motion that continues to the end of the work, i.e., the *background*.

Some new symbols are introduced here to indicate aspects of the voice leading. Solid beams indicate voice leading that is in motion, whereas the broken beam denotes stationary voice leading. White notes indicate structurally important harmonies, whereas black notes are accessory step motion

¹Heinrich Schenker (1868-1935), Austrian theorist, was the initiator of this type of analysis.

linking the white notes. Schenkerian notation does not indicate rhythmic values. The key to this type of analysis is the recognition of step motion moving through harmonies. This is invaluable for developing sight-reading skill, because it simplifies the music and engages a broader scan of the music. It also reveals important aspects of the voice leading, for which it was initially designed.

The entire C Major Prelude is contained in the *Anthology of Musical Structures*. A reduction of the harmonies of the complete prelude is as follows:

The diagram shows a musical score for the C Major Prelude with Schenkerian harmonic analysis. The score is divided into measures 1 through 35. Below the staves, chord symbols and domain labels are provided. Domains are indicated by brackets and labels: I (measures 1-4), V (measures 5-11), ii (measures 12-13), I (measures 14-19), IV (measures 20-21), V (measures 22-31), IV (measures 32-33), V (measure 34), and I (measure 35). The chord symbols include ii, V7, I, vii⁹/ii, ii⁹/I, V7/IV, vii⁹/V, vii⁹/V, V7/IV, ii7/V7, and I. Arrows indicate directed harmonies and chromatic motions.

This analysis should be studied carefully. Compare it to the score of the Prelude in the anthology. Note the barlines are shown here only every four measures. Each chord represents one measure. Every note of the Prelude is accounted for. The white notes are chord tones, and the few black notes are nonharmonic; the one in m24 is passing, in m27 and 30 they are suspensions, and the black bass notes are pedals. Verify the chords for yourself; there are several secondary dominants and leading-tone 7ths. The arrows among the chord symbols show the directed harmonies. The arrows on the staff indicate chromatic notes that have a strong sense of direction toward resolution. Sharp notes are usually secondary leading tones, resolving up, while flat notes are 7ths of chords, which resolve down. These strong chromatic motions signal important changes in the music.

Notice at the bottom of the diagram there is a line labeled “domains”. The first four measures have already been shown to be basically an elaboration of tonic, which is why these measures are shown as a tonic domain. The harmonic domains show that not all the chords are equally important, but some dominate specific segments of the music. These domains are always diatonic chords and normally fall into one of four types: tonic, dominant, subdominant, or supertonic. The second domain is dominant, which extends from measures 7-11. Notice that the F<’s are chromatic notes that “point to” this domain.

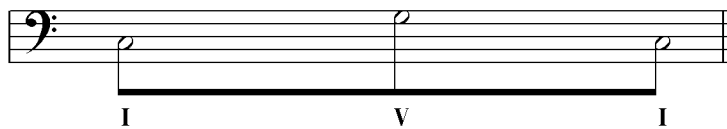
The next domain is supertonic in measures 12-13. Once more, the chromatic notes in m12 point to this domain. Measures 14-19 return to the tonic domain; the strongest indicators are the return to B> (leading tone) and the A=, which is the 7th of the vii⁹ chord. Measures 20-21 emphasize the subdominant due to its secondary dominant. Measures 22-31 center on the dominant, which is emphasized as a pedal in the bass. The F< in m22 functions as the leading tone of G and the A= in m23 functions as a downward leaning note to G.

Measures 32-33 focus on the subdominant. Measure 34 has its own domain, the dominant, as does the last chord, tonic. It is typical in this kind of analysis to regard the final cadential chords as important domain chords.

If one knows the rhythmic figuration that runs through this prelude, one can play the entire piece from this diagrammatic analysis. Therefore, by grasping this structure, the prelude is readily

comprehended and played. This makes it quite powerful for sight reading as well as for understanding the tonal structure.

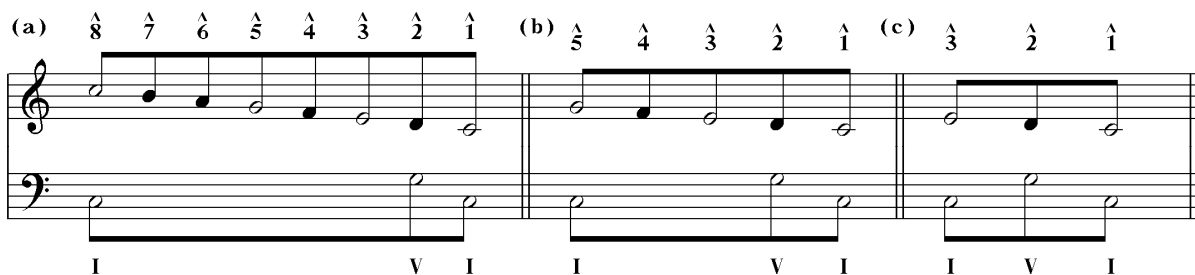
There is an even simpler version to facilitate reading. Let's consider some common factors that are known about tonal music. Firstly, nearly all such music begins and ends in the same key, which we call the tonic. Secondly, nearly all tonal music then diverges away from the tonic in the middle of the piece, often to the dominant key or to some other related key, but most often it will go to the dominant. In fact, this is the plan in binary and sonata form. From this we can deduce a common plan for most tonal music, namely I V I. Now this itself, in the final analysis reduces to simply tonic. Therefore, all tonal music finally reduces to just tonic, since it starts there and it ends there. That is why we can state that a particular piece of music is "in the key of" whatever it may be. This is enormously simplifying. It means that all such music is really just an elaboration of the tonic chord, and the most common divergence is to the dominant. So, the basic harmonic scheme is I V I, with the roots of these chords in the bass. Schenker called this the *Bassbrechung*, or bass arpeggiation. which we can illustrate as follows:



Another Schenker discovery is that there are only three structural soprano lines, which could be called schemas. All of these lines descend, and since they start with a tonic chord, they can only have one of three possible starting notes: 1, 3, or 5. However, almost all of tonal music ends with structural 1 (tonic) in the soprano.



These were called the *Urlinie*, or fundamental lines, by Schenker. When an *Urlinie* is combined with the *Bassbrechung*, the result is the *Ursatz*, or primordial structure. Note that the *Ursatz* is an abstraction and is not the same as the background. There are three possible *Ursatz*:



Schenker believed these to be the only three possible abstract structures in tonal composition. Whether this is so has yet to be proven, but it has been shown that a large number of compositions share these basic structures.

The first diagram of the C major prelude can be regarded as a preliminary foreground. The following is the foreground analysis.

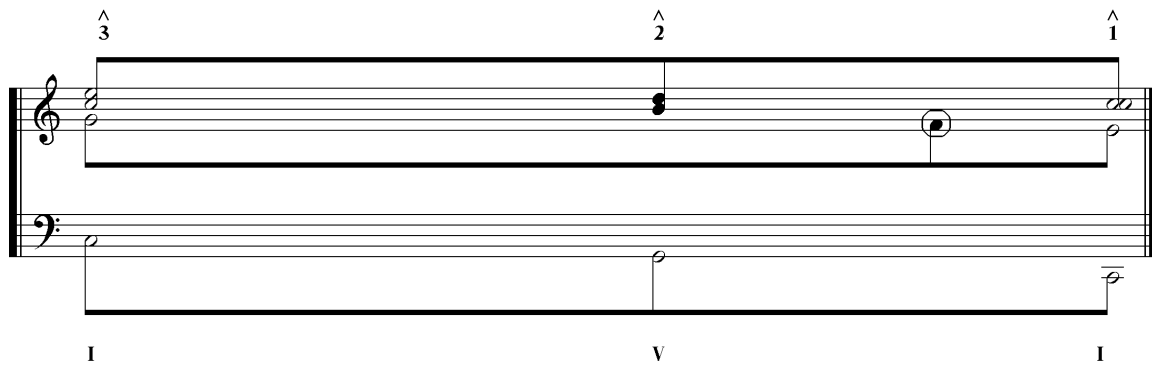
domain: I V ii I IV V IV V I

Once more, the black notes indicate nonharmonic step motion between the harmonic domains. Repeating notes of the preliminary graph have been deleted, and beams indicate the voice leading within each domain. These are all step motions except in some cases where the bass has a solely harmonic function. (These are shown as white notes with flags.) The slurs show the voice leading between domains, which is once again stepwise. This graph shows the horizontal connections and directed motion. Otherwise, this graph is identical to the first.

Between this foreground analysis and the background, a middleground analysis shows the link between the two, revealing more of the basic structure and less of the details than the foreground.

I V I

This is the graph on the cover of this book. Note the bass line descending an octave to m20. Then it moves to the dominant via an incomplete neighbor (F). Finally, in m35 it returns to tonic. The soprano line starts on 3 (E) and descends stepwise to C in m16, moves up to D in m24 and returns to C at the end. Thus, it is a fundamentally 3-2-1 *Urlinie*. Inner voices are also shown on this graph, all of which move stepwise, mostly descending. From 24 to 34 the soprano rises through an arpeggiated dominant, thus transferring the register from D4 to D5. In the background this will be reflected as a stationary D. Thus, we can begin to see the outline of the background in the middleground graph.



A detailed, step-by-step outline showing how these graphs were constructed is contained in the Appendix. Although a detailed study of Schenkerian analysis is beyond the scope of this book, this serves as an introduction to this type of study. It is recommended that the student study the analysis in the appendix. Most of the theory journals today are replete with this type of analysis.

Assignments

1. Read the appendix analysis of the Bach C major Prelude and study the process of the analysis. Think ahead about how to apply this in analyzing the piece in assignment #2.
2. Using a Schenkerian approach, analyze Beethoven Sonata, op.27, no. 1, second movement, the first 40 measures. This piece is in the *Anthology of Musical Structures*. Follow these instructions:
 - a. Construct a foreground verticalization similar to Ex. 1 in the appendix; this will be graph #1. try to fit this graph on two lines of score.
 1. Use second endings only. Place bar lines every 8 bars and measure numbers above each chord.
 2. Use and maintain the top 3 notes of each chord in each clef at the beginning. Most chords will have identical PCS in the bass and treble (3 apiece, 6 total – some are nonharmonic), at least until approaching the cadences; exceptions are in m16 and m40 (measures 16 and 40). Use one note per stem for the soprano and bass parts; alto and tenor voice parts will have two notes per stem. All should maintain their normal stem directions.
 3. Omit successively repeating chords and octave doublings within each clef, except in m40.
 4. Keep the voice leading smooth to simulate the way we hear. To do this, some transposition by octave is necessary:
 - a. Transpose m13-15 up one octave in bass only.
 - b. Transpose treble of m27-36 to fit in the range of G3 to A4. Transpose only the chords that are out of this range.
 - c. Use bass notes in m13-22, but transpose them up one octave.
 - d. Transpose bass notes of m27-28, 31-32, and 35-36 down one octave.
 5. Leaps of a 5th occur in the soprano from m15 to 16, and in the bass in m12-13, 15-16, and 39-40.
 6. Analyze with composite chord symbols. The chord in m17 is a =VII. Some minor

dominants also occur due to melodic descents through the subtonic. Measure 21 should have an implied 3-note chord in the treble that is identical to the one in the bass. Include it.

7. Write chord tones as dotted-half notes (white) and nonharmonic tones as black (quarters). Nonharmonic notes are: B=, A in m6-9; A> in 13; F and E> in 16; A> and G in 32. (The chord in 32-35 is a iv6)

8. Changes of register occur in m36-37 in the treble only. (The C4 in the alto m36 is transferred to C5 in m37. The alto G3 also transfers up to A4.

b. With the information on graph #1, construct a preliminary foreground graph with no beams or slurs. This should be a separate graph, which will be graph #2.

1. See if you can determine chord domains yourself before proceeding any further. See appendix for instructions on determining domains. Then check the following.

2. The foreground domains are: m1-13 is tonic (I), m14-16 is dominant, m17-20 is subdominant, m21-27 is tonic, m28-39 is dominant, and m40 is tonic. Write these below the staff with brackets showing their inclusive measures (see Ex 2 in appendix if necessary).

3. Omit the time signature and write all domain notes as half notes (white) and all non-domain notes as black (quarters).

4. All immediate note repetitions should be erased in each part.

5. Some voice parts will move in parallel octaves; these are nonessential doublings. Erase one of the parts, normally an inner voice part which is less important. However, do not delete notes that show a separate, unique line. The result should reduce to 3-4 voice parts.

6. Examine and study each part. Can you determine the general direction of each part?

c. Construct the Foreground graph (graph #3). This should have the same notes as #2, but beams and slurs are added to show the voice leading.

1. Beam together all the notes within each part, domain by domain. Put the score on one line if possible.

2. Use slurs to show voice leading *between* domains. All black notes should be accounted for with step motion to or from white notes, filling in step motion between the domain notes. These are “nonharmonics”, some of which are called “incomplete neighbors”.

3. See if you can determine the basic upper line, i.e., the *Urlinie*, and the basic bass line, the *Bassbrechung*. Outline the middleground domains. There are only three.

d. Construct the Middleground graph, (graph # 4).

1. Reduce to 3 lines: soprano, bass and alto. The soprano has 2 “interruptions” (m16 and 25). The first time, the soprano line descends from G to D (m1-15). In m17-24 it descends again,

but from G to E. Finally, in 25-40 it descends from G to C, with alto notes “reaching over” the soprano near the end.

2. Use white notes (half-notes) for domain notes only. Black quarters should fill in the gaps between domain notes with step motion. Beam the notes within each part in the dominant domain only. Use slurs to show other voice leading.

3. The soprano from m1-32 can be shown as a simple octave transfer from G5 to G4. Likewise, the alto is an octave transfer from E5 to E4. And, the bass transfers from C4 to C3 over nearly the same span (m1-27). The rest (28-40) should show the step descent of soprano, alto and bass, from V to I. The *Umlinie* is: 5 4 3 2 1.

e. Construct a background graph (#5) with only a tonic domain; only tonic notes should be white.

1. Reduce the octave transfers to one note per part (use lower note). Eliminate the alto line. The result should be the background.

2. Black notes should move stepwise between the white domain notes.

3. The Background should resemble one of the *Ursatz* schemes, but will not be identical with it.

2. Writing Simple Melodies

Our melodies will be restricted to major and minor tonalities. Thus, the tonal vocabulary will conform to the major and minor scales and the chords that function within those tonalities. In minor keys we will be using only the harmonic and melodic minor, with the melodic form prevailing in the construction of melodies.

Generally, the rules for voice leading and part writing are the same as those used in the text, *The Structure of Diatonic Music*. A review of this text may be in order before proceeding. Some voice leading rules are relaxed here under special conditions that will be outlined below. For example, leaps greater than a perfect 4th and perfect 5th will now be permitted as long as restrictions regarding their use are observed.

Since our goal here is to write comprehensibly, we can say that lines composed of predominantly smooth motion are facile and intelligible while those that predominantly leap are difficult to unintelligible. Thus, this is one attribute that helps us to distinguish an intelligible melody from one that is not. However, a melody can be so smooth that it becomes dull. Such is the case with the following examples.



Ex 3a is the smoothest possible motion, no motion. It is also the dullest. Ex 3b moves stepwise back and forth between two notes, while 3c does the same thing with three notes. All of these examples are melodically monotonous, static, and boring. Thus, while smoothness is an ingredient of good melodies, it does not guarantee interest. Interesting melodies must, therefore, balance smooth motion with leaps². Step motion tends to be more predominant in vocal music than in instrumental music.

Interesting melodies also have a definite shape and a sense of direction. Always sing and play what you write. Good melodies are singable. There are four possible directional shapes: 1. arch, 2. down line, 3. up line, and 4. bowl, listed in order of their frequency in music. In addition there is one possible ancillary shape, the wave. The wave is not used alone, but always in combination with one of the other shapes.



²While the lines shown are not interesting, they could validly be used as accompaniments. However, melody is considered a focus here.

This tune has the shape of an arch, which is the most common shape of melody. It creates tension on the rise, climaxes in the middle, and falls gently to a resolution. On the downward part the line is combined with the wave. The following tune from Verdi's *Rigoletto* consists of two phrases, each of which is a falling line.

Verdi: "Caro nome", *Rigoletto* (1851)
moderato



An upward line increases tension as it rises.



Finally, a bowl-shaped tune releases energy with its initial downward inertia, only to build it up again with a subsequent rise and return to the initial pitch. This shape is relatively rare.

Schubert: Sonata in A, op. 120 (piano)



When writing a melodic phrase be sure that it has one of these clearly defined shapes. The climax, or highest pitch, in any of these shapes should only occur once, at least in prominent rhythmic positions. Additionally, the line should have direction, a goal. This goal is most often either tonic or dominant, as can be seen in the above examples.

Although step motion and small leaps are relatively unrestricted in use, large leaps must be used with caution, and certain restrictions are best to observe. (Small leaps are those of a 3rd or perfect 4th. Large leaps are intervals larger than the perfect 4th.) Large leaps of a 6th or more should be balanced with motion in the opposite direction unless a single chord is being arpeggiated. Such leaps are generally followed by step motion. Successive leaps of a 4th are not good unless the notes conform to the harmony (in most cases they cannot). Additionally, successive leaps of a 5th or larger in the same direction should be avoided.



In general, refrain from excessive motion in one direction, but instead unidirectional motion should be balanced with opposite motion, and abrupt halts of rhythm should be avoided within each phrase,

since the phrase should have continuity and a directed goal. The range of a single phrase should not exceed a 12th (octave plus 5th).

Too many different rhythms (note durations) result in rhythmic chaos. There should be rhythmic corroboration and consistency. Repeating notes should not be used in counterpoint exercises. They only occur in music if they are a part of a motive or a text is being used. Note repetitions tend to infer vocalized syllables.

Although leaps of 6ths, 7ths, and octaves may be used judiciously, augmented intervals and major 7ths generally should not. Some tritones may be used occasionally if they are resolved properly. Specifically, the diminished 5th that occurs between scale degrees 4 down to 7 (Ex1) must resolve by step up to the tonic. The diminished 5th moving from 7 up to 4 (Ex2) must resolve down to 3. The augmented 4th from 4 up to 7 (Ex3) and 7 down to 4 (Ex4) should be avoided entirely for the time being, as should augmented intervals in general.

1. okay 2. okay 3. wrong 4. wrong 5. okay (ornamented resolutions)

The notation shows five measures of music on a treble clef staff in C major. Measure 1: D4-G4 (d5) resolving to G4-F#4 (a4). Measure 2: G4-F#4 (a4) resolving to F#4-E4 (m3). Measure 3: F#4-E4 (m3) resolving to E4-D4 (m2). Measure 4: E4-D4 (m2) resolving to D4-C4 (m2). Measure 5: D4-C4 (m2) resolving to C4-B3 (m2) with a grace note on C4.

A melody may imply parallel 5ths or octaves by arpeggiating them. These should not be used in succession and are considered the same as forbidden parallels (Ex c, below).

The notation shows a single melodic line on a treble clef staff. It consists of several measures of eighth notes, with some measures containing arpeggiated chords that imply parallel intervals.

The melodic minor should be used in constructing lines, not the harmonic or natural forms (Exs d-e). A series of one-directional step motions, especially in fast notes, followed by a leap in the same direction should also be avoided (Ex b).

Good melodies also have the property of a clear harmonic background. Step motion normally fills in the gap between chord tones as passing tones. The determination of the harmonic background is dependent upon several factors, most of which are a matter of sorting out the more important notes from those that play a lesser role. Contributing factors are:

1. arpeggiation of chords in the melody
2. metrical accents
3. longer note durations
4. note recurrences
5. first and last notes, especially last (cadences)
6. highest and lowest notes (outlines)
7. harmonic tones as opposed to nonharmonic

In some melodies the harmonic background is obvious because chords are arpeggiated within the melody.

Bach: WTC1, Prelude No. 9

Melodies that move stepwise use other criteria (see above list) to establish harmonic background:

The harmonic background should be a progression, analyzed as primary chords unless a complete secondary chord is outlined. Avoid retrogressions. Successive leaps in the same direction should outline a single chord. Nonharmonic tones should be used and resolved properly. Most melodies eventually return to tonic, as in the above example, but this may be delayed with intervening phrases before the final tonic is reached.

Motives in Melody

Most good melodies are made up of smaller units, called motives. Motives are basic recurring melodic units of one to several notes. They serve as the “building blocks” of melodic structure. Most of the time they are less than two measures in length, and are often just a few notes. The motive is the basic cell or building block from which most melodies are constructed. It may be defined as the smallest distinctive, recurring musical figure in a composition. Contrapuntal melodies are more motivic than most other melodies. For a basic review of the motive, consult the *Music Fundamentals* text on form. Most motives have two aspects: rhythm and contour.

One of the most common motives in music, and which is found often in the music of Bach, is the four-note pickup motive, here abbreviated PM. It consists of four notes beginning off the beat or on a weak beat and leading to the next beat or to a strong beat. The motion is often but not always stepwise.

(a) Bach, Invention 1 (b) Bach, Violin Chaconne in d (c) Beethoven, Sonata, Op. 27/1/iv (d) Beethoven, Piano Sonata Op. 31/2/iii

(e) Brahms, Waltz, Op. 39/13 (f) Op. 39/13 (g) Mendelssohn, Op. 7 (h) Mahler, “Ich bin der Welt abhanden...”

The PM is universal and can be found throughout the literature of music. Often it is truncated to two

or three notes.

Standard transformation (variation) operatives are used on an original or prime (P) motive and melody. These include: 1. transposition, 2. inversion (P played upside down), 3. retrograde (P played backwards) 3. retrograde inversion (P played upside down and backwards), 4. augmentation (slower note values) 5. diminution (faster note values), 6. interpolation (inserting notes in the middle or beginning), 7. extension (adding notes at the end), 8. permutation (changing the order of notes), 9. elision (subtracting notes from middle), 10. fragmentation (a segment of the motive), 11. rhythm change, 12. pitch change, 13. turns, 14. syncopation, 15. interval expansion or contraction, etc. This is not an complete list of possible variation operatives. Normally these operations are used one at a time in order to maintain the identity of the motive, but cumulatively they can lead to fairly remote metamorphoses. This accumulation of transformations, Arnold Schoenberg³ called “developing variation”. Developing variations are found in the music of Liszt, Brahms, and many composers who followed them.

On the following page is a single motive with many transformations. Study these carefully, sing and play them. At the bottom of the page is a melody from Mozart’s opera, *Don Giovanni*, which is composed of this motive and various transformations of it.

³Arnold Schoenberg (1874-1951), famous Austrian composer.

Compound Melody: Implied Counterpoint

Some melodies consist of more than one voice condensed into what seems to be a single line. This creates *implied counterpoint* and a *compound melody*.



(a)

(b)

The image shows two musical staves in 3/4 time, D minor. Staff (a) shows a single melodic line with eighth and sixteenth notes. Staff (b) shows the same line broken into two voices: the upper voice follows the original melody, and the lower voice provides a harmonic accompaniment of eighth notes, creating the illusion of two voices.

This line is from Bach's *chaconne* from the d minor *Sonata* for solo violin (see the anthology for a complete score). The first (a) example is shown as written, while the second shows how the line breaks up into two voices, one answering the other. This creates the illusion of two voices within a single line. In some cases three or more voices may be implied. The following is taken from the same piece and shows how three voices are projected into a single line. Ex(a) shows the original notation, and (b) reveals the three voices.



(a)

(b)

The image shows two musical staves in 3/4 time, D minor. Staff (a) shows a single melodic line with eighth and sixteenth notes, starting at measure 50. Staff (b) shows the same line broken into three voices: the upper voice follows the original melody, the middle voice provides a harmonic accompaniment of eighth notes, and the lower voice provides a harmonic accompaniment of eighth notes, creating the illusion of three voices.

Summary-Outline for Melody Writing

General Information

- L Relative importance of notes is determined by:
 - Duration, high and low points, first and last notes, metric position, accents, harmonic versus nonharmonic, tonal hierarchy
- L All good melodies imply a harmonic background.
- L Implied counterpoint can occur within a single line.
- L Skips imply chords; step motion fills in the gaps between chord tones.
- L Melodic considerations favor the melodic minor; minor melody mostly conforms to this scale.
- L Harmonic aspects favor the harmonic minor scale, as when a line outlines chords;
however, these scales are not completely independent. They interact and their forms may change according to the demands of the linear and harmonic structure.

Voice-leading should:

- L move mostly stepwise, with occasional leaps for variety (average = 2 leaps per 8 quarter notes)
- L have a definite shape and direction . . . should not meander aimlessly
 1. arch is most common
 2. Straight line up or down
 3. Bowl
 4. Wave (only combined with one of the other shapes)
- L phrase should climax only once
- L outline chords or a harmonic background and have harmonic direction
- L observe rules of harmonic progression
- L successive leaps in the same direction should outline chords; otherwise avoid
- L use the melodic minor unless outlining chords
- L resolve nonharmonic tones properly
- L use rhythmic or melodic motives to unify the melodic structure. May sequence up to 3 times.
- L move in opposite direction of large leaps unless arpeggiating a single chord
- L resolve leaps of d5, d7 or m7 with step motion opposite of leap
- L consider tendency of scale degrees
- L end (final cadence) on tonic approached by step
- L Avoid:
 - augmented intervals, major 7ths, and leaps larger than an octave
 - range in excess of a 12th within a phrase
 - successive large leaps in the same direction
 - series of fast step motions followed by a leap in the same direction
 - many differing rhythmic patterns
 - abrupt halts in rhythm within a phrase
 - more than 3 sequential motions in succession
 - stepwise harmonic and natural minor; however, the harmonic form is used for leaps
 - multiple climaxes within a phrase (normally on strong beats)
 - circular melodies
 - two leaps of a 4th, 5th, or larger in same direction
 - implied parallel 5ths or octaves
 - excessive motion in one direction
 - repeating notes unless motivic

3. Introduction to Counterpoint

Why Study Counterpoint?

Virtually none of present day music is monophonic anymore. Thus, any modern musician must learn to cope with polyphony. Counterpoint and harmony are the two essential dimensions of all polyphonic music.

Counterpoint is the one dimension that most distinguishes a large body of Western music. Until the twentieth century most non-western music was monophonic, and today many countries of the world are struggling to preserve what remains of this historic tradition. However, there is little question that the polyphonic tradition of Western civilization has spread throughout the world, and that there is little music anywhere anymore that is not polyphonic, adopting the principles of Western harmony and counterpoint. To ignore the study of counterpoint is to be ignorant of the craft that has made this music. The combination of lines creates a musical fabric of subtlety and richness that many believe has never been achieved or surpassed in any other way.

Perhaps the primary reason for studying counterpoint is to awaken and sharpen the student's sensitivity to this important and sophisticated musical dimension, to bring an awareness and understanding of the result of combining simultaneous lines. Some of this can be achieved through analysis, but only with the actual practice, or writing and playing, can this dimension be truly understood and become alive.

Brief History of Contrapuntal Theory

Around 1300 the word *contrapuntus* first appeared in describing the writing of note against note, or point against point. Various tracts appeared in the fourteenth century which codified methods for writing more than one melody at a time. These have been attributed, albeit doubtfully, to various authors, including Johannes de Garlandia (*Introduction to Counterpoint*), Johannes Muris (*Art of Counterpoint*), and Philippe de Vitry. In 1412, the Italian theorist Prodromus de Beldemandis (*Treatise of Counterpoint*) explained that the meaning of "counterpoint" had changed from note-against-note to writing two different melodies that are combined simultaneously. Both horizontal (melodic) and vertical (harmonic) dimensions had to be considered.

In the fifteenth century, Johannes Tinctoris first described a systematic approach to writing counterpoint. He called this *species counterpoint*. The first species, which he called *simple counterpoint* was writing point against point (note against note). The second species he called *diminished* or *florid counterpoint*, where a diversity of rhythm occurs between the parts. He considered this to be superior to the first species because of its diversity. In writing about point against point it is clear that Tinctoris had mathematical concepts in mind. Yet he also considered memory of linear motion to be a critical aspect of good contrapuntal writing.

The Italian composer and theorist, Gioseffo Zarlino (1517-1590), who discovered the major and minor triads, wrote a treatise on Renaissance counterpoint in which he made a distinction between

the old Medieval practice and the new art (*ars nova*) and in which he admonished composers to consider the overall structure of a composition, rather than short term note for note writing.

In the seventeenth century, Claudio Monteverdi described a new distinction between the new style, or “second practice” (Baroque), and the old “*stile antico*” of the Renaissance. He issued a challenge for a new theoretical model, which wasn’t realized until the eighteenth century, with Jean Philippe Rameau, who made the distinction between “harmony” and “counterpoint”. The other challenge was to formulate a didactic system for writing counterpoint. This was accomplished by Johann Joseph Fux (1660-1741).

In 1725 Fux wrote a treatise on counterpoint, which he titled *Gradus ad Parnassum (Steps to Parnassus)*. His theory was based upon the modal contrapuntal music of the sixteenth century, primarily the sacred vocal music of Giovanni Pierluigi da Palestrina (1525-1594), an Italian counter-reformation composer. Fux thereby chose to ignore the more modern work of his contemporaries, such as Bach and Handel, in formulating his contrapuntal steps to Parnassus. Fux’s treatise was written in the Socratic style and intended as a composition text. Although other texts on counterpoint preceded Fux, none had a greater influence. It became the standard theoretical basis of counterpoint instruction even up until today, although it had many shortcomings when compared to the actual practice of counterpoint in Palestrina’s music and Fux even violated his own contrapuntal principles in his own music. Even so, his model was used by Haydn and Mozart.

In the eighteenth century Fux’s approach was modified by Johann Georg Albrechtsberger (1736-1809). He adapted Fux’s species approach to major and minor tonalities. It was in this form that it reached most composers of the nineteenth and twentieth centuries. Many famous composers learned their craft from this, including Beethoven, Chopin, Liszt, and Brahms. Beethoven himself studied counterpoint with Albrechtsberger.

Fux’s theory has since become antiquated by developments starting with J.S. Bach and continuing to the present. New practices occurred largely in the harmonic domain; i.e., Fux’s conception of consonance and dissonance needed to be revised and expanded. A crisis occurred in the teaching of counterpoint in the nineteenth century when this widening gap between the theory and the practice became commonly acknowledged. The shortcomings of Fux’s system were lamented.

Eventually, new methods tried to encompass the harmonic practice of Bach’s time (eighteenth century). But the revisions were unsystematic compromises, basically patching up Fux’s theory. Recently, a kind of reverse Schenkerian approach has been applied to contrapuntal theory, using a series of diminutions to a basic *Ursatz* framework.

In the twentieth century, Schoenberg used a modified Fuxian model during his teaching career at UCLA. In his own *Preliminary Exercises in Counterpoint*, he stated:

To base the teaching of counterpoint on Palestrina is as stupid as to base the teaching of medicine on Aesculapius. Nothing could be more remote from contemporary ideas and structure than the style of this composer . . . On the other hand, there is no greater perfection in music than Bach!

Already in the works of J.S. Bach many of the older laws are denied. Not only does he rarely use modes, preferring instead to write in major and minor, but, additionally, his treatment of dissonances is changed and is much freer. He uses seventh-chords and ninth-chords, extends the concept of nonharmonic tones,

and alters many other restrictions due a new notion of melody. Later, in Brahms, and especially in Wagner, this change went as far as possible, and the face of music changed so radically that it seemed for a time anachronistic to study counterpoint at all.⁴

It is significant to note that although Schoenberg was writing some of his most radical new twelve-tone compositions at the time, nowhere in his teaching do we find him instructing students in modern serial composition. Instead, he taught traditional counterpoint, not because he had to, but because he believed in its benefits.

The student should recognize that study and writing according to any theoretical model is an abstraction, not a reality. It assumes that working in a disciplined, rule-based environment, that at least closely conforms to the practice, is a worthwhile educational venture. Last, but not least, it should not be forgotten that many of our most revered composers have learned their craft through the study of this model. Today, courses in counterpoint are normally divided into sixteenth century (Palestrina) and eighteenth century (Bach), although neither one really simulates the writing of any real composer.

The current text is intended as an introductory study of counterpoint and is modeled on the practice of Bach and later composers, and although the species approach is still used, it is supplemented here to bring it into agreement with modern tonal practice. Although it does not go into atonal, dodecaphonic, polytonal, or other specialized twentieth century practices, a section on dissonant counterpoint is included. While the current text retains the systematic merits of the Fuxian system, a new approach is used which incorporates Schenkerian diminution, resulting in a more realistic model for modern counterpoint.

⁴Schoenberg, Arnold. *Preliminary Exercises in Counterpoint*, prefaces, St. Martins Press, NY, 1963.

4. Basic Contrapuntal Technique

As an introduction to counterpoint, our study will be confined to two parts. This is the foundation of all other more complex forms, such as those in three and four parts.

In writing counterpoint it is important to remember that the goal is to write equal, independent voices. That is, one part should not dominate another, and they should, in so far as possible, be independent. Thus, one part should not become an accompaniment to the other, but both should share equally in interest. When dealing with the primary parameters of pitch and time, voices can be independent in pitch by favoring contrary motion. The types of motion from most independent to least are: 1. contrary, 2. oblique, 3. similar, 4. parallel. So, parallel motion is least desirable (although necessary) in counterpoint. In fact, restrictions are placed upon the use of parallel motion (aside from parallel 5ths and octaves). Voices can be independent in time by having different rhythms.

Although the voices should be horizontally independent, they are harmonically dependent. They are governed by the use of consonance and dissonance. The definition of consonance and dissonance as “pretty versus ugly” or “pleasing versus harsh” are totally useless, subjective, and naive. Therefore, these terms need to be defined to make them useful to us. Dissonance is any chord, sound, or interval that has restrictions (rules) placed upon its use, whereas consonance is sound that is relatively free in its use. Notice the word “relatively”. This means that there are no absolute, universal consonances or dissonances.

A Historical Note about Consonance and Dissonance

The use of C&D (consonance and dissonance) has changed historically. In the ancient world only the perfect intervals were consonant, i.e., P8, P1, P5, P4. All the others were considered dissonant. This was also true in the Middle Ages when the perfect intervals were given religious significance. They were Godly, holy, and pure, whereas the others were simply called “imperfect”. (The names for major and minor intervals did not yet exist.) Thus, they were “impure”, tainted, and corrupted in some way, i.e., dissonant. The major 3rd and 6th, for examples, were dissonant and initially were not used in early organum (early polyphony), which moved in parallel octaves, 4ths, and 5ths. Today, 3rds and 6ths are the sweetest and most consonant intervals as compared to the perfect 4th or 5th, which now are considered empty or “hollow” when heard alone. Even if we compare the treatment of dissonance in the Renaissance with that of, say, Bach, there is a profound difference. Renaissance counterpoint was governed by the interval, but Bach’s is governed by chords. Thus, we find many instances of practice in Bach’s music that actually violate the principles that governed Renaissance counterpoint.

Further, nineteenth century counterpoint found in Brahms, Wagner, and Strauss, etc., is freer in its treatment of dissonance than in any previous music. In the twentieth century we have heard composers speak about the “emancipation of dissonance”, where the distinction is either blurred or erased. Recent studies have shown, however, that even in the most “dissonant” music of this century a new logic replaces the old; i.e., there are still consonances and dissonances but they have been “redefined” and used in new ways.

Consonance is metaphorically resolute, static, and restful, whereas dissonance is dynamic, moving, and energetic. Both are necessary. Without the dynamic power of dissonance, consonance becomes boring and impotent. Throughout history it seems that as people became used to the old dissonances they lost their energy and strength. Composers, then compensated by “pumping up” the dissonances, and treating the old dissonances as consonances. Thus, it seems to some who are used to older styles, that modern music is too dissonant. This has been the perennial complaint that never abated throughout history. Many people in Beethoven’s time thought his music was too dissonant as he invented new chords and did not resolve others. Beethoven’s Ninth Symphony was considered so incomprehensible by some that they blamed it on his deafness. He was totally deaf when he wrote it.

So, we can see that the concept of C&D is not rigid or universal. This is to say nothing of the music found in other cultures, where even microtones are frequently found. How, then, can we agree on what is consonant and dissonant? The model we will use here is the one initially based on Fux’s model. This is called species counterpoint, which is the only systematic approach available to date that remains constant in training musicians around the world. It is also the one used by many of the great masters of the past and present. It is expanded here to agree with later practice, but twentieth century “dissonant counterpoint” cannot be embraced, because its principles are contrary to those of this tradition. However, a chapter is included at the end of this text that explains some of the basic tenets of dissonant counterpoint to impart some knowledge of that practice.

Species Counterpoint

Consonance and Dissonance (Harmonic)

Not all harmonic (vertical) consonances are equal; i.e., some are more restricted than others. The freest intervals are 3rds and 6ths, and these are, therefore, the most consonant. Next are perfect octaves, unisons, followed by perfect 5ths. The perfect 4th is to be considered a mild dissonance but it sometimes crosses over to the consonant category, which will be explained later.

The order of dissonant intervals from the mildest to the strongest is: perfect 4th, minor 7th, tritones, major 7th, major 2nd, minor 2nd. In general all other augmented and diminished intervals are also in this class.

General Principles

Each line must be good in itself, i.e., following the principles of good melody outlined earlier. The lines must be independent; i.e., avoid too much parallel motion. Parallel and consecutive 5ths and octaves are strictly forbidden. Additionally, in two part counterpoint, parallel 4ths are forbidden, and a 7th to octave has the same effect as parallel octaves. Unequal 5ths and octaves are also not permitted in two parts, with the exception, in some species, of perfect to diminished 5ths moving stepwise (often at a cadence) – the diminished 5th must, however, resolve by contracting to a 3rd. Avoid direct octaves and 5ths unless the soprano moves stepwise. Even these should be reserved for cadences.

An X in the following examples indicates an error.

(a) X //8 (b) X //5 (c) okay in 2:1 (d) X D8 (e) okay (cadence) D8 (f) okay (at cadence) D5 D8 (g) okay d5 3

There should be rhythmic and motivic corroboration within and among the lines. Consonant intervals should predominate. Octaves and unisons are used only in contrary or oblique motion or in cadential doublings. Avoid moving from perfect 5th to octave in similar motion, or vice versa, except at cadences. Do not double leading tones, chromatic tones, or other active tones (such as 7ths of chords), and no crossed voices. Voices should not separate by more than two octaves plus 5th, and most of the counterpoint should occur within about two octaves. All intervals should be notated with numbers.

Since counterpoint collaborates with harmony, principles of harmonic progression should be

(a) good (b) poor (not a cadence) (c) poor D5 (d) dlt (e) chromatic X (f) crossed X

observed. It may seem that full chords do not occur since there are only two notes at a time. However, a chord is often implied or completed by notes that are adjacent in time to a given interval; i.e., notes that are in close proximity, especially if they are part of an arpeggiation. Nevertheless, there are cases in which the harmony may be ambiguous in two part counterpoint. In C major, the notes G B are unambiguous, clearly implying dominant. But, why couldn't it be the mediant, E G B? An implied chord with the root present takes precedence over one in which the root is absent. So G B cannot be called mediant in C major unless E occurs in close proximity as part of an arpeggiation. Additionally, when the chord in question is ambiguous and can be either a primary or secondary chord, the primary chord is usually the one that the ear favors. In the final analysis, several factors may play a part in the identity of the harmony, and one must rely on good hearing. Due to these uncertainties, restrictions on chord inversions and progressions are less clear-cut than they are in four parts. It is okay, for example, to have (implied) diminished chords in any position as long as the tritone is resolved properly. However, occasional uncertainties should not be used as excuses for retrogressions, etc. If there is doubt, ask your instructor. To allow for doubt, one retrogression per exercise will be permitted as long as it is labeled ² and checked for effectiveness.

In any case, composite chord symbols should accompany the exercises. If only one pitch-class occurs, as in an octave, it may not be possible to choose a chord symbol, in which case a question mark will suffice. On the other hand, an octave on the tonic at the end or beginning almost certainly implies a tonic chord. So-called "ambiguous intervals" are often not so, especially when preceded or followed by notes that complete the chord.

5. First Species, 1:1, The Foundation

First species is note against note counterpoint, or one-to-one. It has the strictest set of rules and is the most important species. Mastery of first species part-writing is important before moving on to other species.

L Oblique motion should not be used, because a repeating (or held) note is no longer 1:1, but becomes 2:1 (two notes against one) with respect to part-writing. In general, rests, note repetitions and ties are not included in first species.

Thirds and Sixths

L 3rds and 6ths are the most common intervals in first species counterpoint. They may be used freely, but too many parallel 3rds or 6ths defeats the independence of parts. Therefore:

L No more than three parallel 3rds or 6ths are permitted in short exercises. Often four or more may occur in longer music, but they are restricted in short exercises in order to concentrate the counterpoint. Remember that the longer a string of parallel intervals continues the less the counterpoint and independence.

Dissonant Intervals

L No 2nds may be used in 1:1.

L A diminished 5th resolves stepwise by contracting to a 3rd.

L An augmented 4th resolves stepwise by expanding to a 6th.

L 7ths are approached and left by step. If a leap is involved, the notes must be part of a single chord.

L A minor or major 7th is part of a 7th chord where the soprano (the 7th) resolves down by step. The bass should move up by 4th, down by 5th, or up by 2nd to resolve the chord implied.

L Diminished 7ths should resolve by stepwise contraction to a perfect 5th.

L Perfect 4ths, if used, must be approached and left stepwise in contrary motion, or they can imply a cadential six-four in a cadence. Since there are very few opportunities for such conditions in first species, perfect 4ths are rare.

L Chromatically altered intervals must resolve correctly according to the principles of the chord they imply; diminished intervals contract and augmented intervals expand.

Fifths, Octaves, and Unisons

L No parallel or consecutive octaves, unisons, 5ths, 4ths, or unequal 5ths may be used.

L Octaves are normally approached and left in contrary step motion. If a leap is involved, the notes must be part of a single chord.

L Octaves and unisons may occur at cadences, at phrase beginnings, or in contrary motion within the phrase.

L Avoid direct 5ths and octaves except at cadences.

L The *horn fifth* is an acceptable 5th in *similar motion* (direct 5th); it proceeds from a 6th to a 5th and then a 3rd, or vice versa (see the example that follows). At a cadence, it can lead to an octave in

similar motion (D5 to D8).

L Do not begin with or cadence on 5ths. A perfect 5th may be used only in contrary step motion, unless it is a horn 5th or is at a cadence; 5ths should be surrounded by 3rds and 6ths.

L Cadences should be on unisons, octaves, 3rds, or 6ths only.

L Use no *consecutive* perfect, mixed intervals in a row, except at cadences.

In the following examples an X indicates an error.

(a) oblique X (b) //3 X (c) //6 X (d) okay d5 (e) okay m7 (f) good horn 5ths (g) good unison

Examples of First Species

Example 1 is a typical example of first species in a major key. Note the use of composite chord symbols in addition to the interval numbers. These should be used to check the implied harmonic progression. Although the rules regarding progressions are treated more freely in two-part counterpoint, they must still be taken into account so as not to unintentionally weaken the key.

Most of the intervals are 3rds and 6ths. All octaves except the cadential perfect octave occur in contrary motion. The augmented 4th in measure 3 resolves by expanding stepwise to a 6th.

In example 2 three perfect octaves occur. It is common but not necessary to begin and end with an octave. The octave in the second measure is used in contrary motion. The diminished 5th in the first measure resolves by stepwise contraction to a 3rd.

Example 3 in c minor, starting with a unison and ends with an octave. The score consists of two staves. The upper staff has notes: C4, D4, E4, F4, G4, A4, B4, C5. The lower staff has notes: C3, B2, A2, G2, F2, E2, D2, C2. Figured bass notation is provided below the staves.

c: i vii^o i v6 iv6 ? iv i6 vii^o I V7 #vi^o V6 I

Example 3, in c minor, starts with a unison and ends with an octave. These are the common intervals with which to begin and end. Notice the 7th in the third measure. It outlines the dominant 7th (G7). A diminished 5th follows the 7th; thus, there are two dissonant intervals in a row, which is permissible if they both resolve properly. The 5th contracts stepwise to a 3rd and the 7th resolves down by step. The melodic minor is used in the bass at the end. In measure 2 the A= in the soprano is not raised even though it seems to be ascending. This is because the A= leaps to C, outlining part of a chord (iv). It conforms to the harmonic minor for this reason (leaps outline chords). The bass in measures 1-2 descends stepwise and uses the descending form of the melodic minor. The B> on the second beat is ascending, so it is raised. Note also the use of predominantly contrary motion and 3rds and 6ths.

Example 4 also uses mostly contrary motion and 3rds and 6ths. Notice the final cadential octave approached in contrary motion. The octave in the second measure is also approached and left in contrary motion. An augmented 4th follows it, which expands stepwise to a 6th, resolving properly.

Example 4 in a minor, starting with a unison and ends with an octave. The score consists of two staves. The upper staff has notes: A4, B4, C5, D5, E5, F5, G5, A5. The lower staff has notes: A3, G3, F3, E3, D3, C3, B2, A2. Figured bass notation is provided below the staves.

a: i V6 i iv6 V7 i6 iv ? iv I⁶ V6 i

Example 5, also in minor, has several octaves and a diminished 5th. The diminished 5th is treated like a passing tone in the soprano and resolves by contracting to the 3rd. The large leaps are followed by contrary step motion, except the bass in the last measure leaps down a 6th and then down a 3rd, or, as an alternative, up a 3rd. Either is possible since these leaps outline a single chord. The downward 6th does finally resolve by contrary motion stepwise to the last D, although it is embellished with the E. Note also the string of four 6ths in the last two measures. This may seem to be a violation of the maximum of three parallel 6ths, but only two are parallel 6ths. The others move in contrary motion. The restriction applies only to a string of parallel 6ths or 3rds.

6 8 3 3 8 d5 3 6 6 #3 6 3 8 6 3 6 6 6 #6 8

d: i6 ? i V7 i ii₆ V III₄ ? III iv i₆ V₆ V i

Chromaticism in First Species

Chromatic notes may be used in first species if they are accounted for as:

1. secondary leading tones which resolve up by a semitone.
2. 7ths of chords, such as secondary-dominant 7ths or leading-tone 7ths, which resolve down by step.
3. distinguishing notes of other chromatic chords, such as Neapolitans and augmented 6ths, that resolve properly.
4. chromatic passing or neighbor tones which resolve properly by step.

Cross relations should be avoided when using chromaticism. A cross relation consists of a note that is immediately preceded or followed by the same pc chromatically altered in another voice. In order to imply a chromatic chord at least one other note of the chord must be present in proximity to the chromatic tone and preferably simultaneous with it. However, it is not considered a cross relation if the unaltered note immediately precedes the chromatically altered note in the same voice, as in the second measure that follows.

chromatic 1:1

3 #6 8 3 3 8 6 3 1

C: I vii₆/V V V/vi IV ? IV₆ V I

Study these examples and play them.

cptF1-1

6 8 a4 6 8 3 6 3 8 3

C: ii₆ V₄/vi C: vi₆ V₆/iii iii vi₆ ii V I
 A: iv₆ V V₄ i₆

Modulation may also occur in first species, and in fact the last example may be analyzed as a

modulation from A minor to C major as shown as an alternative.

6. Florid Counterpoint

Second Species, 2:1

In second species the rhythm is two notes against one. All species after the first are freer in their treatment of dissonance, but harmony still regulates what is acceptable.

Additional considerations in 2:1 are:

1. No parallel or consecutive 5ths or octaves should occur from beat to beat or on consecutive strong beats. Also, they should not occur on consecutive off-beats in a sequence.
2. Oblique motion may be used, but not repeating notes. However, repeating notes may be used in the preliminary 1:1 before converting to 2:1 (see below).
3. Step motion from P5 to d5 is permitted if the d5 resolves by contraction to a 3rd. However, d5 to P5 is considered parallel 5ths.
4. Any dissonant interval may occur as long as the notes are accounted for as a standard harmonic or nonharmonic tones and resolved properly. Most dissonant intervals should occur off the beat.
5. Consecutive, mixed perfect intervals may be used as long as they are part of a chord arpeggiation.
6. 7th to octave motion is permitted in 2:1.

Otherwise, there is little difference between first and second species restrictions.

Second species is best thought of as a diminution of first species; i.e., notes are added that embellish the essential counterpoint of first species. These “embellishments” may either be consonant skips that outline chords or they may be nonharmonic tones. For an illustration, let’s recall an earlier example in first species.

The image shows a musical score for a first species counterpoint in C major. It consists of two staves: a treble staff and a bass staff, both in common time. The treble staff contains the notes G4, A4, B4, C5, B4, A4, G4. The bass staff contains the notes C3, F2, G2, A2, B2, C3. Below the treble staff, intervals are labeled: 8, d5, 3, 8, 3, 3, 5, d5, 3. Below the bass staff, Roman numerals are labeled: C: I, vii°, I₄, ii, V₅, I.

By embellishing this with quarter notes we can create a Schenkerian diminution in 2:1.

cptA2-1

C: V7 I⁶/₄ ii V⁶/₅ I

Note the dissonant intervals in the diminution. Here all the added notes are off the beat, but this need not be so. The second note in the first measure acts as an escape tone, and although the diminished 5th doesn't seem to resolve as it should, its resolution is to the 3rd simply delayed by the consonant 6th. The same is so in measure 4. The first 4th in measure 2 is passing, and the second one is a "consonant 4th" that is simply part of the arpeggiated C chord. In measure 3 the two 2nds are passing. In embellishing the 1:1 counterpoint one must be careful that the non-chord tones are accounted for as standard nonharmonic tones, and they should be labeled as such. It must also be re-checked for the possible introduction of parallel 5ths and octaves.

Here is another of our earlier examples.

cptC1-1

a: i V6 i iv6 V7 i6 iv ? iv i⁶/₄ V6 i

And, here it is converted to second species.

cptC2-1

a: i vii^o i iv6 i⁶/₄ V²/₂ i6 iv ? ii^o i⁶/₄ V6 i

Note the succession of intervals from measures 1-2: 5 8 4 a4 2, which seems to be an improbable sequence. Three consecutive, mixed perfect intervals are okay due to the 5 8 4 chord arpeggiation. The 4th is a *consonant 4th*, i.e., one that is part of a chord arpeggiation, in this case the tonic. The a4 is part of the dominant 7th and resolves properly to a 6th (delayed). The consecutive a4 to 2nd seems most improbable, but the E in the soprano is the root of the V chord and needs no special treatment, even though the interval with the bass is a 2nd. (Both the E and G are harmonic notes.) The D in the bass, however, is the 7th and must resolve down by step, which it does. Therefore, this seemingly improbable series of intervals is justified by the chords that are outlined.

Any dissonant interval may be used if the notes are part of the underlying chord.

The dissonant 4th at the end of the second measure is an escape tone. In measure 3 the E= is a chromatic passing tone. Note that it is actually sounding with an E> in the bass, but it is not a cross relation. The a4 of the second beat expands to the 6th stepwise as expected, and the next 4th is consonant.

Now let's recall one of our chromatic examples in first species.

cptF1-1

C: ii6 V_{4/2}/vi vi6 V₆/iii iii vi6 ii V I

Through chromatic diminution we get:

cptF2-1

C: ii6 ? V_{4/2}/vi vi6 V₇/iii iii vi₆ ii V₇ I

Be sure to play all of these examples. Note here that the F< of measure 2 has been inserted on the beat rather than off. This is simply a chromatic passing tone embellishing the following G< and does not change the harmonic implication of the secondary dominant. Most of the other added notes are chromatic passing tones. In the penultimate measure an F accented passing tone, which is actually part of the dominant 7th chord, has been added before the G. Although there are many accidentals here, the whole example is in C major.

On the following page are some examples of diminutions which can be used to convert 1:1 to 2:1, 3:1, and 4:1. However, even the 4:1 figures can be used in 2:1. Can you determine how? The first measure shows the initial figure, which is a stepwise descending line. The other figures are various diminutions up through 4:1 of the original. These should be studied carefully. This is not an exhaustive list. Many others can be created. As the number of notes of diminution increases, the possibilities multiply. The ones shown are some of the most common figurations. Notice the corresponding ascending figures on the second half of the page. All these figures should be played and sung while listening for the basic line in each diminution.

Notice that there are no figures here that outline the interval of a 5th, as in Alberti bass. This is because such a figure would create parallel 5ths if used in a sequence. However, a 5th outlined-figure may be used in combination with another type. In fact, all of the above figures may be used in various combinations rather than alone. The possibilities are almost infinite.

Try inventing or improvising some of your own diminutions of ascending and descending lines. Consult various scores of Bach, Beethoven, etc., for more possibilities.

Often, composers elaborate on a basic line in both parts. The following is from Beethoven's Sonata for piano, Op. 26, last movement. Note the basic structure of the counterpoint at the top of the figure (parallel 3rds), the two part diminution below that, and the "unfolding" by arpeggiation at the bottom. The diminution is accomplished by stepwise "escape tones" in the soprano and consonant skips in the bass.

Beethoven, Op.26/iv
basic structure

The image displays three musical staves in G major, 2/4 time, illustrating the evolution of a rhythmic figure. The first staff, labeled "basic structure", shows a sequence of six chords, each marked with a "3" below it, representing a triplet of eighth notes. The second staff, labeled "2 part diminution", shows the same chords with a soprano line and a bass line. The soprano line uses stepwise "escape tones" (e.g., G-A-B-A-G), while the bass line uses consonant skips (e.g., G-F-E-D-C). The third staff, labeled "arpeggiation (actual score)", shows the figure as it appears in the original score, with the chords arpeggiated.

3 **3** **3** **3** **3** **3**

2 part diminution

3 **6** **3** **6** **3** **6** **3** **6** **3** **6** **3**

arpeggiation (actual score)

3:1 and 4:1 (Third Species) Counterpoint

The principles for 3:1 and 4:1 are not much different from those for 2:1. The most important consideration regards the possible introduction of parallel 5ths or octaves. Generally, they are questionable if they occur in any part of one beat to the first interval of the next beat. But, they are admissible occasionally if they are audibly hidden (perhaps by occurring on the weakest part of a beat as part of a fast note figuration). In any case, it is best to avoid them.

With these faster notes more care needs to be given to possible unidirectional step motion followed by leaping in the same direction. There is also more of a tendency for the line to meander aimlessly or to contain too many different patterns. Faster motion demands more motivic unity and constancy of figuration. Sequences are helpful. More care needs to be given to resolving nonharmonic tones.

Examples of converting 1:1 to 3:1 and 4:1 were shown earlier. Review these. The following is an example from Bach's *Well Tempered Clavier, Book I, Prelude 6*. The top score shows the basic plan for the first measure in 1:1. Below that is Bach's diminution of this line of 3rds.

$\text{♩} = 80$

The image displays four systems of musical notation for the first measure of Bach's *Well-Tempered Clavier, Book I, Prelude 6*. The first system shows a basic plan for the first measure in 1:1, with a tempo marking of quarter note = 80. The second system shows Bach's diminution of this line of 3rds. The third and fourth systems show further variations and fingerings for the first measure. Fingerings are indicated by numbers 1-5 below the notes. The bass line is consistently simple, often using a single finger (1) for the first measure.

At the bottom is a chart of the second measure counterpoint, and Bach's diminution appears just above it. There are two other voices that are not shown on the chart. Can you find them?

NOTE: Constant arpeggiated chords, like those shown above, are not recommended in short

exercises.

Fourth Species: Syncopation

This species is characterized by ties, suspensions, and notes occurring off the beat. An example is from Bach's *Two Part Inventions*. The top score is a reduction showing direct first species in the first three measures. The second and third scores show Bach's realization. Notice that the notes are displaced to the right by one sixteenth in the first three measures, thus creating the fourth species with the bass "leading". In measures 5-7 it is the bass that is displaced by the same amount; thus, the soprano leads. The result is a series of suspensions as shown by the intervals. Measures 5-7 are a retrograde of measures 1-3.

♩ = 70

The musical score consists of three systems, each with a treble and bass staff. The first system (measures 1-4) shows a reduction of first species in the first three measures, followed by Bach's realization where notes are displaced to the right by one sixteenth. The second system (measures 5-8) shows the retrograde of measures 1-3, with notes displaced to the left by one sixteenth. Fingerings and intervals are indicated below the notes.

The common dissonant suspensions are the: 7-6, 4-3, 2-3, 9-8, 7-8, and the less common 4-5. In addition there is a "consonant suspension", the reversible 6-5. In the 7-6, 4-3, 9-8, and 6-5 suspensions the bass leads, and the soprano falls. In the 2-3, 7-8, 5-6, and 4-5, the soprano leads, and the bass falls. All these suspension types occur in the above example. You should locate these, circle them and study their formation. Although many of these suspensions lack ties, suspensions are often notated with ties.

The leaders should not be switched in these types of suspensions. The Bach example employs what is called a "chain suspension", i.e., a succession of suspensions. They more normally occur isolated.

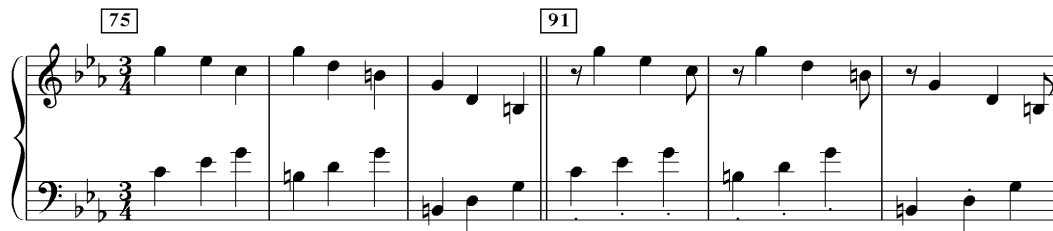
Here are some of the most common types of suspensions:



Notice how each of these is prepared and resolved, and try writing some of your own.

Syncopation does not require suspensions, however. The following shows a consonant type of syncopated counterpoint.

Beethoven, Sonata Op. 27/1/i i



Both of these examples are from the second movement of the piano sonata, Op.27, No.1, by Beethoven. The first three measures are the corresponding score measures 75-77, and the last three are measures 91-93. Beethoven transforms 75-77 by syncopating the counterpoint in 91-93. He did this by simply displacing the soprano line by one eighth. Can you create a similar syncopation by displacing the bass? These are consonant syncopations. Each measure arpeggiates a single chord. Other examples of syncopation may be found in the anthology, such as in Bach's *Little Prelude* No. 7.

Fifth Species, Free Counterpoint

Most real contrapuntal music is not restricted to one species but is a mixture of different types. This is called "free counterpoint", or "fifth species". Moreover, a composer is not obliged to always write contrapuntally. So, in real music counterpoint is often mixed or alternated with homophony or non-contrapuntal textures. Such a mix is not "fifth species", however. For examples of fifth species counterpoint, consult the anthology, especially Bach's keyboard works, such as the *Prelude* and *Fughetta*, BWV 900, the *Partita* No. 3 *Fantasia*, the *Passacaglia* by Handel, and the *Prelude XX* from WTC 1, by Bach. All of these works utilize a combination of species types, but they are mostly combinations of first and second species, at least on the surface.

7. Form Generating Techniques

Imitative Counterpoint

One of the most important form generating techniques in counterpoint is called *imitation*. Often, this begins with a solo, in one voice, that is immediately imitated in another voice. So, the imitation is sometimes called an “answer”. This technique is common in Renaissance sacred polyphony, canons, rounds, fugues, and a number of other lesser known contrapuntal forms, such as the *caccia*. The following example is the beginning of Bach's first two-part *Invention*. Notice the subject stated by itself at the beginning, followed by the answer in the bass.

The image shows a musical score for the beginning of Bach's first two-part *Invention*. It consists of two staves: a treble clef staff (top) and a bass clef staff (bottom). The time signature is common time (C). The key signature has one sharp (F#). The score is divided into two measures, labeled '1' and '2'. In measure 1, the treble staff plays a melodic line starting with a quarter rest, followed by a series of eighth notes: C4, D4, E4, F#4, G4, A4, B4, C5. This is labeled 'subject'. In measure 2, the bass staff plays a similar melodic line starting with a quarter rest, followed by eighth notes: C3, D3, E3, F#3, G3, A3, B3, C4. This is labeled 'answer'. The treble staff continues with a melodic line in measure 2, starting with a quarter rest, followed by eighth notes: D4, E4, F#4, G4, A4, B4, C5, D5. The bass staff continues with a melodic line in measure 2, starting with a quarter rest, followed by eighth notes: D3, E3, F#3, G3, A3, B3, C4, D4. The score is annotated with 'subject' and 'answer' brackets, and measure numbers '1' and '2' in boxes.

When the answer is stated, the first voice has 1:2 counterpoint against it. Note that the turning eighth note figure that follows the subject in the treble is not imitated; i.e., the imitation is broken after the initial statement. If the imitation was continuous, i.e., without a break, it would be a canon. In the second measure the imitation resumes, this time on the dominant. But it, too, is not continuous. The entire invention is in the *Anthology of Music*.

There are two possible types of answers: *real* and *tonal*. A real answer imitates the exact intervals of the subject. A tonal answer is one that is altered slightly in order to direct the tonality (real answers often modulate) or to prevent a dissonant entry of the answer. The answer of the above example is real but does not modulate, and generally this is the case with imitation at the octave. Often, however, the answer in a *fugue* is tonal. The next example shows a tonal answer at the beginning of a fugue by J.K.F. Fischer.

Fugue by J.K. Fischer (1665-1746)

The image shows a musical score for a fugue by J.K. Fischer, measures 2 through 12. The score is written in 3/4 time and B-flat major. It consists of two staves: a treble clef staff and a bass clef staff. Measures 2-4 show the subject in the bass. Measure 5 shows the answer in the treble, starting on F. Measures 6-12 show the subject and answer in various parts, including a canon in the bass starting in measure 9.

Notice the answer in the treble in measure 5-9 starts with an F rather than a G (fugue answers are normally given on the dominant). This creates an interval of a third that answers the 2nd at the beginning of the subject, C to D. This change is to prevent a dissonant entry of the answer in measure 5. A treble G would have created a dissonant 2nd with the bass. Otherwise, the answer would be real. The technique for writing imitative counterpoint is discussed in the next section on canons.

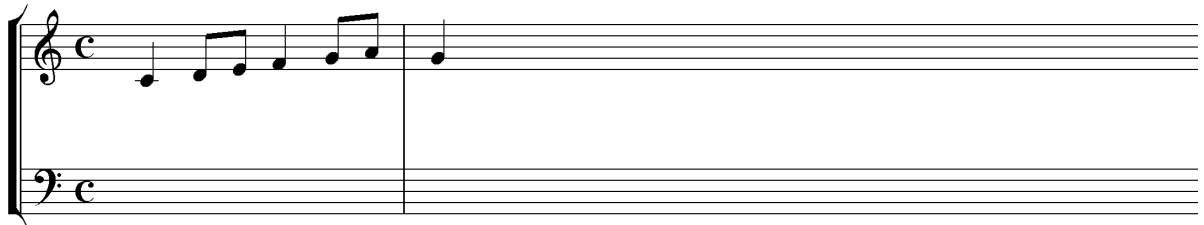
S

The word “canon” means “rule” or “law”. Thus, a musical canon follows a strict procedure from which the composer does not waver. A canon is a complete piece of music that uses *continuous imitative counterpoint*. The primary difference between canon and general imitative technique is that in a canon the imitation is continuous, i.e., it never breaks, except perhaps to accommodate the final cadence. Therefore, imitation is used to create the form of the canon. A canon is not defined by sections as in binary or ternary forms, but by a special contrapuntal scheme, namely continuous imitation.

Canons are classified in various ways. One important parameter is the “interval of imitation”, which actually consists of two variables, pitch and time. Here, we will refer to these as the *pitch interval* and the *time interval*. The pitch interval in the Bach *Invention No. 1* is an octave whereas in Fischer’s fugue it is a 4th. The time interval in Bach’s *Invention No. 1* is half a measure, whereas in Fischer’s fugue it is 4 measures.

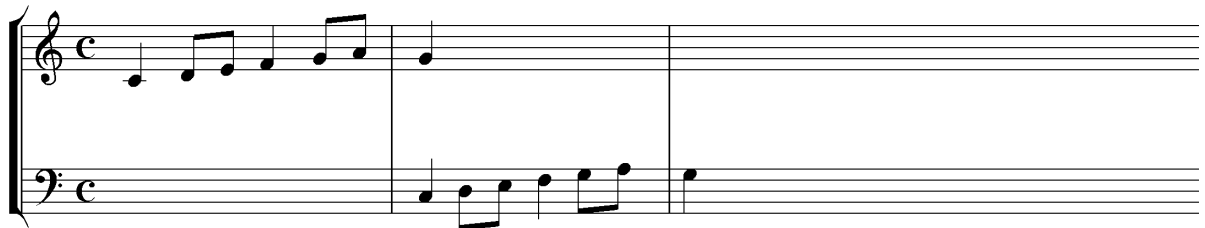
How to Write a Canon

The technique for writing a canon is quite simple. First one writes the initial subject as a solo for the time interval.



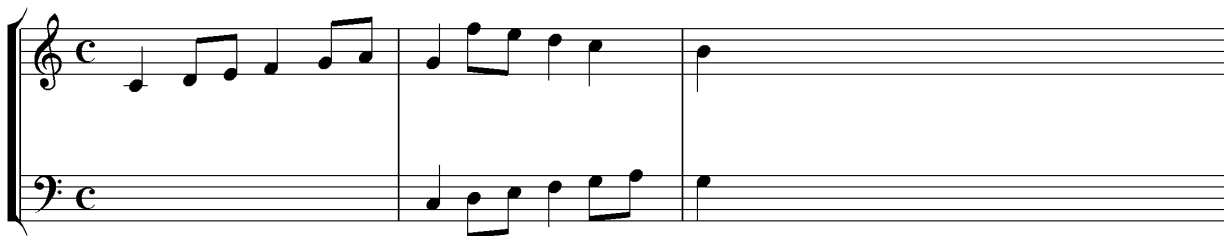
A musical score in treble and bass clefs, common time (C). The treble clef contains a single melodic line: a quarter note C4, followed by eighth notes D4, E4, F4, G4, and a quarter note A4. The bass clef is empty.

Next, write the answer at the pitch interval desired in the second voice. In this case, we've chosen the octave, which is the most common and facile type.



The same musical score as above, but now the bass clef contains a melodic line: a quarter note C3, followed by eighth notes D3, E3, F3, G3, and a quarter note A3. This represents the answer in the octave below the subject.

Then write counterpoint against the answer.



The musical score from the previous step, but now the treble clef contains a counterpoint line: a quarter note C4, followed by eighth notes D4, E4, F4, G4, and a quarter note A4. This counterpoint line is written against the answer in the bass clef.

Repeat this process any number of times to generate a whole canon. The following diagram shows the steps involved. Other types of canons, described in the next section, can be written with this same technique, including upright canons at any interval, canons in contrary motion and in augmentation, etc.



A musical score in treble and bass clefs, common time (C), illustrating the steps of a canon. The treble clef contains the initial subject (step 1) and subsequent entries (steps 3, 5, 7). The bass clef contains the answer (step 2) and subsequent entries (steps 4, 6). Brackets above and below the staves label each entry as 'step 1' through 'step 7'.

The even-numbered steps are imitations, while the odd-numbered ones are new counterpoint. Imitative counterpoint uses the same procedure, except that after the initial imitation the music may break into free counterpoint.

Special Types of Canons

Rounds, or “Perpetual Canons”

A *round* is a circular canon; i.e., it is a canon that repeats indefinitely. Such a canon links its conclusion with its commencement. Rounds are usually vocal compositions, such as *Frere Jacques* and “Row, Row, Row Your Boat”. They are less sophisticated in construction than other canons and often use only one harmony, namely tonic. Such rounds are sometimes found in African and Indonesian polyphony.

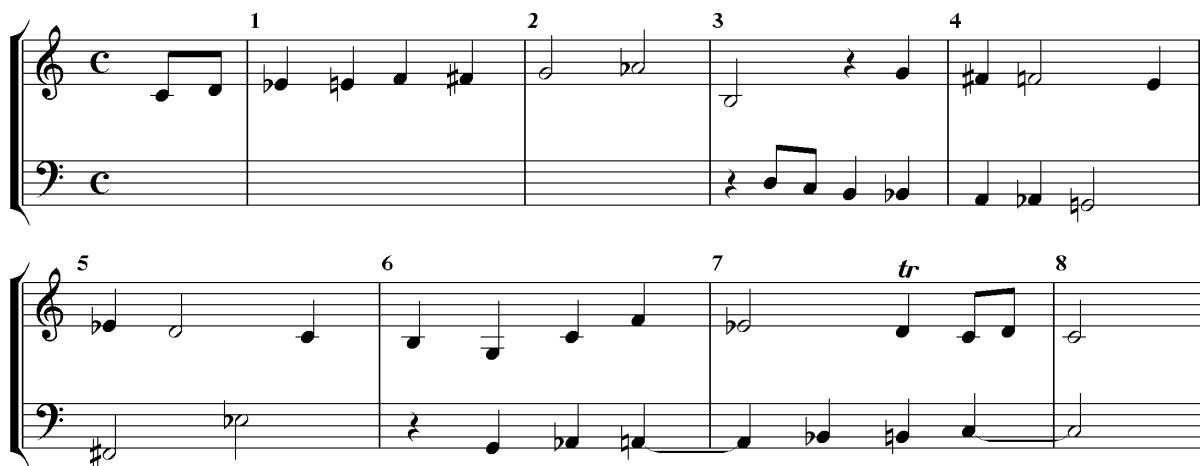
Canons in Contrary Motion

In addition to canons at various intervals, there are types that are determined by special contrapuntal techniques, such as inversion or retrograde motion. In a canon of inversion, or *contrary motion*, the answering voice takes the inversion of the subject as its answer. The following example is a canon in contrary motion from Bach’s *A Musical Offering*. This particular canon is also called an *enigma canon* or *puzzle canon*. Bach and some of his contemporaries delighted in creating musical puzzles of this sort. Only the beginning of the canon is shown.

Quaerendo invenietis (“seek and ye shall find”)



Bach: canon in contrary motion at the 7th, from *A Musical Offering*



This doesn’t look like a canon but only one voice. But, notice the two clefs, one of which is upside down, the bass clef. This tells us that one voice begins on C and plays what is written in the C clef.

The other reads the bass clef upside down so that the upward line becomes a downward line. It begins on D. But what is the time interval between the voices? It is not specified. Instead, Bach wrote the cryptic instruction, “seek and ye shall find”. Thus one must find the time interval by trial and there may be more than one solution. The score shown above is a partial realization of this canon; only the beginning is shown.

Crab Canons

Another special type of canon is the *crab canon* or *canon cancrizans*. In this construction one voice plays from the beginning to the end, while a second plays simultaneously from the end to the beginning. Here is one notated as an enigma canon from Bach’s *A Musical Offering*.

Bach: puzzle crab canon from *The Musical Offering*



This is the entire canon. Notice the backwards C clef, key signature, and time signature at the end. If one can read backwards both instruments can play the canon from this score. Otherwise, the realization is shown on the next page.

To write a canon of this type may seem to be a daunting task, but the basic procedure is fairly simple. Write two-part counterpoint to the midpoint. Then graft the second half onto the end of the first, beginning with the last note and working backwards, i.e., in retrograde.

In writing such a canon it is revealing to examine what happens to the counterpoint. Firstly, in Bach’s crab canon, bar 10 is the axis. At this point, both voices reverse their lines, but in invertible counterpoint (at the unison!). There is a considerable amount of voice crossing in this canon, but Bach manages, nevertheless, to control the resulting harmony.

Notice the harmonic analysis. After bar 10, the harmonic progression is reversed, yet Bach manages to avoid retrogressions. Most of the harmonic scheme is a retrograde, but measure 14 does not exactly fit the retrograde scheme. Compare it to measure 5, its retrograde parallel. Although measure 14 has a subdominant chord (preceded by its leading-tone chord), measure 5 does not. Can you

determine why this is? However, the leading-tone seventh of iv in measure 5 is followed by a supertonic, which is a logical substitute for the subdominant.

In order to avoid retrogressions, Bach uses no third progressions, such as I to vi, which, if reversed would become a retrogression. Therefore, all the root movements are by 2nds, and 4ths involving tonic. This may be a corollary for writing strong crab canons.

A significant conclusion from these observations is that the lines and counterpoint need to clearly outline a harmonic progression in order for such a canon to work properly. Notice that the order of attacking notes are changed after bar 10; e.g., comparing measure 8 with 11, at the beginning of m8 the treble E sounds with a G in the bass, but in m11 the corresponding E sounds with a C. The D in the treble of m8 sounds with a B in the bass, but in m11 the D (bass) sounds with an F in the treble. This is all due to the retrograde motion. Each of measures 8 and 11 consists of two chords, i6 and V4. The first and last notes of the faster figures help to corroborate (outline) each harmony, because the attacking order is reversed after the midpoint. In this case, the intervening notes also corroborate the harmonies.

The Spiral Canon

The spiral canon is a little known, esoteric type of canon in which the canon modulates through a cycle of keys. Thus, it cycles back on itself, but each time it is played in a new key. A spectacular example in the form of an enigma is from Bach's *A Musical Offering*.

The image shows a musical score for the Spiral Canon, consisting of four systems of two staves each. The top staff of each system is in a soprano clef (C1) and the bottom staff is in a bass clef (C2). The time signature is common time (C). The score begins with a repeat sign and a fermata over the first measure. A second measure contains a section symbol (§). The music features complex rhythmic patterns, including sixteenth and thirty-second notes, and various accidentals. The key signature changes from one flat (C minor) to two flats (D minor) in the second system, and continues to change in subsequent systems. The piece concludes with a repeat sign and a trill-like flourish.

The bass and tenor clefs indicate the canon, another “seek and ye shall find” type. At the end is a repeat sign with what looks like a trill notation before it. It is actually a spiral seen from the side. This indicates a repeat with transposition. The first time the canon modulates from c minor to d minor, up a major 2nd. The § (measure 2) indicates the point of repeat and the final cadence. Each time the canon repeats it is transposed up a major second. It takes six repeats to cycle back to the original key, where it would end. In the dedication copy Bach wrote “*Ascendenteque Modulatione ascendat Gloria Regis*” (May the glory of the King rise with the keys).

A partial realization of this fascinating musical structure is shown on the next page.

Canons of Augmentation and Diminution

In a canon of augmentation one of the canonic parts moves at a slower speed than the other. Usually the speed ratio is 2:1. Thus, the augmentation is in notes that are twice as long as the other part. In a canon of diminution a voice imitates at twice the speed of the original statement. This type of imitation is shown in the following example.

Bach: canon in diminution and contrary motion, from *Art of the Fugue* (contrapuntus 6)

Invertible Counterpoint

Invertible counterpoint, sometimes called “double counterpoint” or “convertible counterpoint”, occurs when the content of two voices is switched. This device occurs frequently in music. With two voices the parts would be in the soprano and bass. The simplest way to do this is to transpose one of the parts above or below the other. In the example shown below, the treble part is transposed down an octave, and the bass is transposed up an octave in (a) to get (b). The total transposition, T, is two octaves.

Notice that the intervals become inverted when the voice parts are switched. Octaves invert to octaves or unisons, 3rds invert to 6ths, 6ths invert to 3rds, and 4ths invert to 5ths, etc. Since 3rds and 6ths are equally consonant, there is no problem inverting them; likewise for octaves and unisons. Since 5ths invert to 4ths, however, this was considered a problem by Fux; i.e., the 5th had to be treated as a dissonance in the original in order to invert it correctly. Since our counterpoint is governed more by chords than intervals, this is not normally a problem anymore. But, caution is advisable. Generally, a 5th or 4th should be okay for inversion at the octave if at least one of the voices moves stepwise or if it is part of an arpeggiation of a chord.

The following is an example from Bach's *Invention No. 5*. A complete score is in the anthology. The soprano and bass parts of measures 1-4 are switched in 5-8; i.e., what was in the soprano is switched to the bass, and what was in the bass is switched to the soprano. Additionally, the soprano has been transposed down an 11th, an octave + 4th, and the bass has been transposed up a 12th, an octave + 5th. These numbers (11 & 12) indicate the total transposition, T , of the counterpoint. Specifically, $T = x + y - 1$, where x and y are the transposition displacements of the voices. In this case $T = 11 + 12 - 1$, which is 22, or three octaves. (Octaves = 8, 15, 22, 29, etc.)

Any transposition that is a multiple of the octave is called simply “inversion at the octave”. Therefore, the *Invention No. 5* is an example of invertible counterpoint at the octave. If the separation of the voices ever exceeds T , the voices will cross when inverted, and this should generally be avoided.

Rule: In invertible counterpoint, the voices should not separate by an interval greater than the total interval of transposition, T . Or, in other words, T should be equal to or greater than the largest separation of the voices.

Inversion at the octave is the most common interval of inversion. Others are possible, but are more difficult to achieve successfully. The second-most common intervals for inversion are the 10th and 12th. These are not recommended for beginners.

Inversion at the 10th creates special problems: 3rds invert to octaves (and vs), and 6ths invert to 5ths. this means that no parallel motion may be used (!), because parallel 3rds will become parallel octaves, and parallel 6ths will become parallel 5ths. The story is more complicated than this, though, because the old masters seemed to know secrets about double counterpoint that have not been completely described. For example, there is a passage in Bach's first *Invention* that seems to obliterate these laws.

Each of the two voices in this example has been inverted; i.e., the sixteenth notes of (b) are an inversion of the sixteenths of (a), and the eighths of (b) are an inversion of the eighths of (a). Additionally, the counterpoint in (a) is inverted in (b) by the interval of a 10th; i.e., the sixteenths are transposed up a 2nd, and the eighths are transposed down a 9th ($T = 2 + 9 - 1$). But, the intervals of (a) and (b) are identical! Bach apparently knew that by inverting each of the two voices individually in combination with inverting the counterpoint at the 10th resulted in the same intervals! This means that any counterpoint may be inverted in this manner.

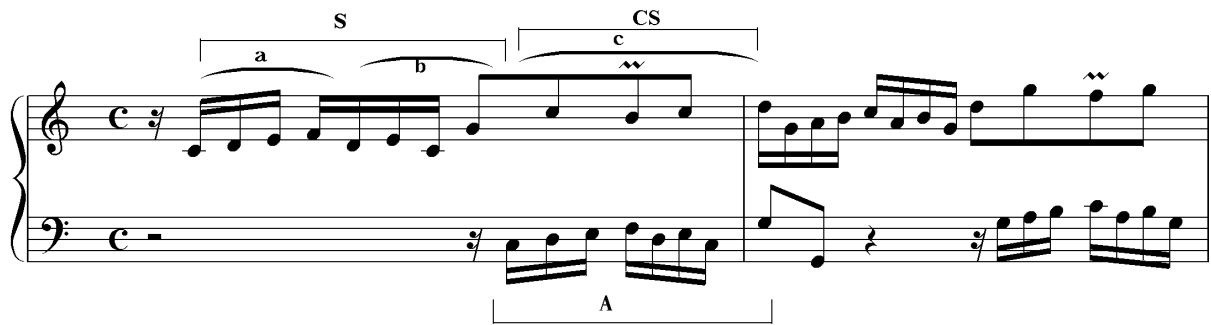
Invertible counterpoint at the 12th also has special problems. Specifically, 6ths invert to 7ths. Therefore, in order to invert at the 12th, the 6th must be treated as a dissonance in the original counterpoint. Parallel motion, then, is limited to 3rds.

8. Writing Short Contrapuntal Pieces

We will discuss three models for contrapuntal pieces: 1. the invention, 2. the ground bass variations, and 3. the binary dance suite movement.



Bach's two-part *Inventions* have served as models of counterpoint for over a hundred years. The invention may be described as a work using imitative counterpoint based entirely on just a few motives. These motives are used to construct a subject and sometimes a countersubject. The rest of the invention, which has a consistent number of voice parts (in this case two) is built from these. The following is from the beginning of Bach's two-part *Invention No. 1*, which is in the anthology.



The three motives are designated a, b, and c. The subject is labeled S, and the countersubject is labeled CS. The subject is stated at the outset of the invention, often, but not always as a solo. This often serves to prompt an imitative statement in the second voice, which is the case here with the answer, A. Most of the inventions start with this kind of imitation. The subject will be restated throughout the piece and is the most important idea.

A countersubject is the counterpoint against the answer when it is stated in the second voice, and recurs throughout the invention in counterpoint with the subject, which is the case in this invention. However, not all inventions have countersubjects. Consult the anthology for a complete score of this invention, and trace the subject, countersubject and the motives throughout the piece. Think about how you might write a similar invention.

Another interesting invention to study is number 8 in F major, also in the anthology. Everything in this invention comes from the subject and countersubject, labeled S and CS in the example.

The image shows a musical score for a three-part setting in 3/4 time, F major. The top staff is the treble clef, and the bottom staff is the bass clef. Brackets above the treble staff label the first measure as 'S' (Subject) and the second and third measures as 'CS' (Countersubject). A bracket below the bass staff labels the first three measures as 'A'. The notation includes various rhythmic values and accidentals.

This invention is a study in invertible counterpoint. Trace the subject and countersubject through the whole invention, and you will see that this is really all the material that Bach used to make the entire piece. The figure in measure 5 is a variant of CS (CSvar), and the cadential figure in measures 8-10 is a variant of S. You should be able to account for every other note in this piece as part of S or CS.

Another invention that is an intensive study of double counterpoint is number 5, also in the anthology.

Ground Bass Variations

The *passacaglia*⁵ or *chaconne* is a special type of *ground bass variations*. Given a bass theme, this is one of the easiest and most unified types of composition to create. Several of these themes are given in the *Resources for Music Theory & Composition* text. To write a set of ground bass variations, first choose and write the theme in the bass, which will occur as a solo the first time. Repeat the bass theme as many times as there are variations. This will create a score framework for the counterpoint. Then write new counterpoint for each recurrence of the theme in the bass.

Examples in the anthology include Purcell's aria from *Dido and Aeneas*, the Handel *Passacaglia*, and the Bach Chaconne from the *Sonata for Solo Violin* in D minor.

The Binary Dance Suite Movement

The Binary Dance Suite Movement occurs in the Partitas, English and French Suites of Bach, as well as many other short keyboard works. It follows the normal binary scheme.

⁵A *passacaglia* is a special type of ground bass variations that is always slow, in a minor key, in triple meter, and the theme recurs in the bass. A *chaconne* is the same except that sometimes a distinction is made for the theme as a harmonic progression, rather than a bass line.

||: 1 2 :||: 3 4 :||

||: A :||: B :||
I V I - - - I

The diagram indicates four phrases, two in the first section (A), and two in the second (B). However, the piece may be double, triple, or quadruple this length. These phrases are often question and answer schemes, coupled to make a period. Thus, there are two periods. Usually, each phrase is 4 measures; so, the total length here is 16 measures. The first 8 measures begins with tonic and moves away by the end of A, normally to the dominant or relative major key. Sometimes, especially in short binaries, the A section may begin and end on tonic. The B section moves back to the tonic, normally by the last phrase. Each of the two sections repeats. This is the same form that was discussed at the end of the *Music Fundamentals* text. It may become a *Rounded Binary* by repeating the second phrase as the fourth (last) phrase. Sometimes the second section of the suite movement is extended to double its normal length, making a total of $8 + 16 = 24$ measures. This form is represented by Bach's Polonaise from the *French Suite No. 6* in the anthology.

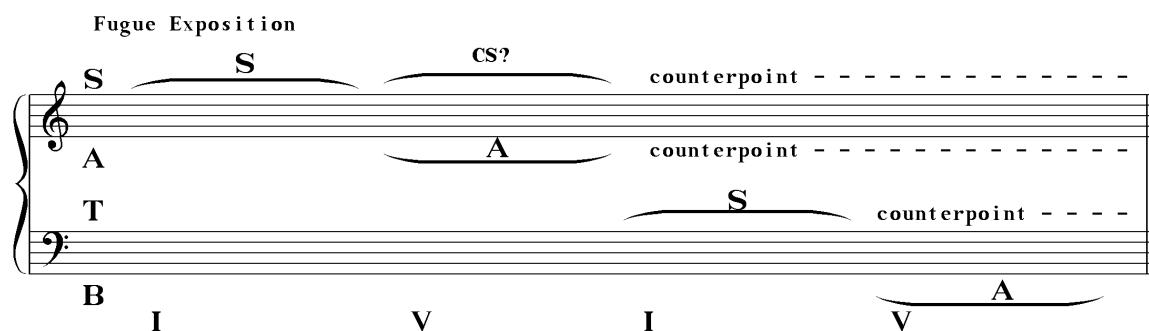
9. Fugue

The technique of fugue writing is beyond the scope of this study⁶, but every educated musician should know what a fugue is and be able to hear and analyze its structure. So, our study here will be limited to a description and analysis.

The fugue is one of the most complex contrapuntal forms of music. It may have two or more voice parts, but is most often for three or four voices. Like the invention, a fugue has a subject and often a countersubject which serve as the primary material from which all other compositional material is generated. All fugues utilize imitative counterpoint and may be divided into sections that can be classified into specific types.

The first section is called the exposition, but should not be confused with a sonata exposition. In a fugal exposition the subject is stated at the outset, most often as a solo (although it is occasionally accompanied by counterpoint). This statement is focused on the tonic key center and may be stated in any voice. When the first voice has completed its statement of the subject, a second voice enters with the subject transposed to the dominant, called the answer. This answer may or may not modulate to the dominant key. If it remains in the tonic key, it is called a tonal answer and will have some alteration of the subject's original intervals. If it is a real answer it will modulate to the dominant key and will have the same intervals as the subject.

In a four-voice fugue, once the second voice completes the answer, a third voice restates the subject back on the tonic. Finally, once the third voice has completed its subject, the fourth and last voice restates the answer. Thus, each of the voices that is to partake in the fugue must state the subject (or answer) in turn in the exposition of a fugue. Once all the voices have stated the subject, the exposition is finished.



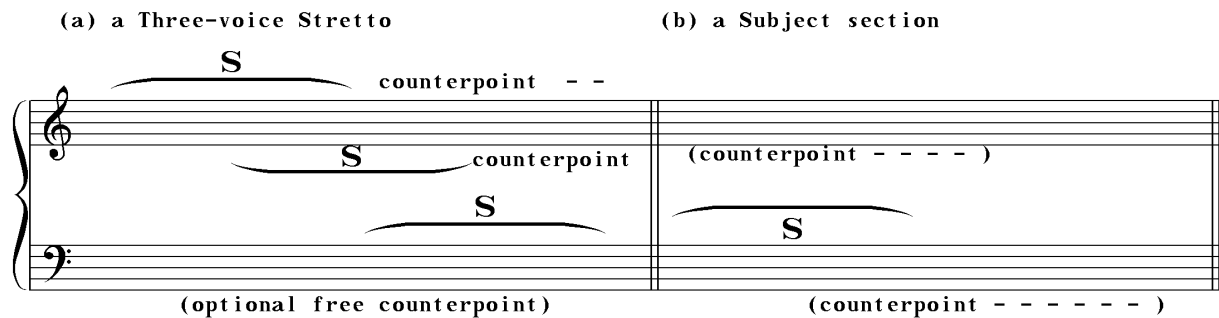
After the exposition, a number of possible formulated sectional types may follow, but not in any prescribed order. These are called the subject section, stretto, counter exposition, and episode. All the conventional melodic transformations may be used, such as inversion, retrograde, augmentation,

⁶for details see: Alfred Mann, *The Study of Fugue*, 1958.

diminution, fragmentation, transposition, etc.

In a subject section, one of the voices restates the subject or any of its transformations, often in a new key, while other voices have counterpoint that is somehow derived from the subject or countersubject.

In a stretto two or more voices state the subject or a segment of it in imitative counterpoint, where one voice interrupts the other so that the statements overlap.



A counter exposition is similar to a restatement of the exposition. The same relative key sequence will follow; i.e., tonic – dominant – tonic – dominant.

The episode is a section that has no complete statement of the subject but is composed of fragments derived from its transformation. Often it has a transitional character, uses sequence, and the number of voices will reduce to two or three.

Sometimes two or three fugues are grafted together, one after another. These are called *double fugues* and *triple fugues*. The second fugue often combines its subject with that of the first fugue. The third fugue, following a similar formula, will often combine all three subjects. The fourth fugue of Bach's *WTC I*, in C< minor, is an example of a triple fugue.

Modern examples of fugues occur in Paul Hindemith's *Ludus Tonalis*, Dimitri Shostakovich's *24 Preludes and Fugues* for piano, Alan Hovhaness's *Prelude and Quadruple Fugue* and his *Symphony No.2 "Mysterious Mountain"*, and the first movement of Bartok's *Music for Strings, Percussion and Celeste*.

A fugato is a fugue that is included as a section of a larger work, such as in a symphony or sonata. A fugato occurs in the central part of the second movement (funeral march) of Beethoven's *Symphony No. 3*. Two fugatos occur as variations in the last movement of Beethoven's *Symphony No. 9*. Can you find them?

Examples of fugues should be studied and played. Some are included in the anthology.

10. Dissonant Counterpoint

Although the intensity of counterpoint in various musical periods changes, the practice of counterpoint has never abated since the thirteenth century. A crisis in traditional harmony occurred at the beginning of the twentieth century. Tertian tonality had been stretched to its limits by composers such as Liszt, Wagner and Strauss in the nineteenth century. In such works as Wagner's opera, *Tristan und Isolde*, there was so much modulation that it was difficult to tell the key or tonic at times. At the beginning of the twentieth century, Debussy, Scriabin, Stravinsky, Ives, and Schoenberg began to explore new dimensions of harmony that were no longer based on the tertian structures that had gone unchallenged for centuries. The hegemony of tonal harmony itself was threatened by a new order. Composers throughout the western world were seeking new ways of revitalizing what had seemed to have become a tired old order that was losing its effectiveness. The old dissonances had, once again, to be "pumped up" if music was not to lose its expressive power. The limit of the old order of counterpoint was reached in passages such as this one from Richard Strauss's *Salome* (1905):



Notice that this passage, intense with chromatic counterpoint, focuses on just one chord, a G Δ 7, until measure 164, where it resolves deceptively to an A major chord.

Some of the answers came from the increasing awareness of different cultures around the world, and the exotic musical structures they used, which seemed fresh and marvelous to many western composers. Debussy began exploring whole tone and exotic scales, parallel fifths and sevenths after hearing music from the Far East. Scriabin explored Russian mysticism and sought a cosmic music. Stravinsky and Bartok were influenced by the folk songs of Eastern Europe and Asia that had been previously little known and unexplored. They began to use their exotic scales and harmonies. Schoenberg, on the other hand, continued in the Viennese tradition, expanding upon the chromatic structures that had been explored by Wagner and Brahms. This adventure brought him to the brink of

the collapse of tonality in music such as *Transfigured Night* and *Pierrot Lunaire*.

Old dissonances began to be treated like consonances, and new dissonances were created to supplant the old. Thus, counterpoint in the twentieth century was, in some ways, revolutionary. In 1930, Charles Seeger, an American composer, proposed a new order of contrapuntal practice that he called “dissonant counterpoint”⁷. In this system, the old dissonances and consonances were effectively switched. That is, 3rds, 6ths, octaves, unisons, 4ths, and 5ths became the restricted (dissonant) intervals, and 7ths, 2nds, tritones, and augmented intervals were unrestricted (consonant) and became the “norm”; 3rds and 6ths had to be resolved like passing tones to 2nds and 7ths, etc. Unisons and octaves were avoided almost entirely. In first species dissonant counterpoint, no 3rds, 6ths, octaves, unisons, 4ths or 5ths were permitted.

Seeger went further and proposed “dissonating” rhythms by disallowing regularity, and dissonating melody by reducing repetition and step motion, using mostly leaps. Much of this theory has actually come into practice. Dissonant rhythms and parallel 7ths are found today in some styles of music, such as jazz. However, predominating leaping melodies and reduction of repetition have only been used extensively in serial 12-tone music. The following example of dissonant counterpoint is from Bartok’s *Mikrokosmos*, No. 132.



This type of counterpoint is encountered in the music of Schoenberg, Bartok, Hindemith, Berg, Webern, Ives, and many contemporary “serialists”. For those who are interested, the following works are suggested for listening and analysis:

Bela Bartok: (1) *Fourth String Quartet*, (2) *Music for Strings, Percussion and Celeste*.

Arnold Schoenberg: (1) *Suite or Septet*, op. 29, (2) *Serenade*, op. 24, *Pierrot Lunaire*

Anton Webern: (1) *Concerto*, op. 24

Igor Stravinsky: *Octet*

Charles Ives: (1) *Three Places in New England*, (2) *Scherzo, Over the Pavements*

Milton Babbitt: *All Set* for Jazz Ensemble

⁷Seeger, Charles, “On Dissonant Counterpoint”, *Modern Music*, v 7, June-July 1930, pp. 25-31.

Appendix A. A Schenkerian Analysis of Bach's C Major Prelude, WTC I.

An Algorithm for Graphic Analysis

In the world of analysis it is rare to find rigorous theoretical systems that provide reasonably consistent and precise results in the hands of analysts of different persuasions. Even set-analysis, which is possibly the most rigorous of all, can give inconsistent results in different hands depending upon how sets are partitioned. Set and Roman-numeral analysis are probably the most widely used analytical systems. They are logical systems in themselves, but when applied, results can and often do vary. This is not quite the same situation for scientific theory, as in physics, where a theory predicts a certain result and, in order to be accepted, skilled experimenters of different persuasions must find the same result, namely that which the theory predicts. This is a part of the scientific method.

It seems improbable that we will ever have a theory of music as rigorous as that of the physical sciences. However, we do demand that the theory predict with reasonable accuracy and reliability. A serious problem with Schenkerian analysis is its lack of consistency, which, in turn, is due to a need for rigor. I am speaking here not of occasionally variant results. Analyses of the same music by different analysts *always* produce different graphs, different results. Reactions to this usually fall into two categories: 1. the theory is wrong and must be scrapped, 2. the theory is incomplete, with partially correct results, but needs more work.

In order to construct accurate and consistent graphs of a Schenkerian type, a logical methodology is necessary. The purpose here is to provide a standard through a method that facilitates the creation of graphs of greater consistency.

A dictionary broadly defines an algorithm as "a step-by-step procedure for solving a problem or accomplishing some end." Although this term is most often connected with computer programs today, it is not meant here to be so confined, since humans can and frequently do carry out step-by-step procedures to solve problems. However, the algorithm does provide the means to create a computer program capable of carrying out such analysis with the current technology. Musical scores can be input with scanners or midi-transcribers and then analyzed with this algorithm.

Theory

Perhaps the most important principle held in the construction of the algorithm is the notion of line as step progression, a fundamental premise of voice leading. Linear motion here is defined as diatonic or chromatic step motion. Therefore, a voice part is located by virtue of its step movement. A search is made for this type of motion and no motion to link a note to another that follows it as a "line". The only justification for this is that cognition of musical lines is apparently conducted through the perception of such step progressions, since these are the smoothest and easiest connections for a mind to make; thus, the step progression becomes very important to the identity of voice parts and lines. This is a basic premise of our algorithm, which searches for these step progressions, and serves the primary aim of this type of analysis, to trace the immediate and large scale voice leading. If step progression cannot be located for a note, then a search is made for

possible octave displacements of the line (octave displacement here includes the leaps of a 7th and 9th). Other leaps, therefore, are excluded from the concept of line. Linear motions are displayed on the graphs with beams and slurs. Beams are also used in the schematic diagrams to show chord arpeggiations.

The bass voice serves not only a linear function, but lays the harmonic foundation. Consequently, the bass may have leaps when no linear path is possible, and such leaps are assumed to have a harmonic, rather than a linear, function. Flagged notes are used to show these movements.

A second premise is that musical structure is perceived on more than the immediate level, *i.e.*, more than on the note-by-note movement that is immediately perceived in hearing a performance of music. This is also a familiar premise behind the use and study of musical forms, such as rondo, sonata, key structure, etc. The premise is here extended into ever longer and larger levels of voice-leading. Thus, it is essential to show these levels in the analytical graphs, and it is this aspect that makes this type of analysis so unique and illuminating. Special symbols are used to display these levels of voice-leading. A black-note is employed to show immediate levels of structure, with an unstemmed one indicating the most immediate level. Flagged black-notes are also used, from the fastest durational value, indicating the most immediate level of structure, to the longest, the eighth note, used for the highest immediate level. Next are unflagged black-notes (quarter notes), the most common note used to display intermediate levels. White notes are used to show remote, or large scale, voice-leading. It should be remembered that these notes do not signify durations and are relative to each diagram in which they occur. The half-note and quarter-note are chosen as the normal starting values for each graph, to which other values may be added when it becomes necessary to show greater levels of structure. In most cases three or four values are sufficient. Dotted notes are not used.

Immediate levels of structure are called foregrounds in our graphs, intermediate levels are called middlegrounds, and remote structures are termed backgrounds. Structure is analyzed starting with the most immediate levels and proceeds to the most remote level, where the only domain is tonic. As we proceed from one diagram to the next, up the structural hierarchy, the immediate levels of structure in the previous graph are deleted, making way for ever larger levels of structure. The immediate levels of each diagram are represented as black-notes in each diagram, and the white notes become the basis for the next higher level of structure.

The third premise is that of the "harmonic domain". This is also known to our perception, but previous to this paper was called by more than one name. Domains are the harmonic functions that a section, or segment, of music focuses upon. It may be a repeating chord, a key center (modulation), a chord emphasized by transient, or secondary dominants, cadential movements, etc. These may also be perceived on different levels, from immediate units of harmonic rhythm, to large scale harmonic regions of the form. There are various ways that these domains may be prolonged. If a modulation occurs, the domain is prolonged by keeping the focus on the new key. A domain may also be prolonged, and thereby dominate a section, by the repetition or arpeggiation (unravelling) of a single chord, perhaps in conjunction with secondary leading-tones or transient dominants. Domains are symbolized with Roman numbers in the graphs. These are often used with horizontal brackets to show the length of a prolongation.

The Choice of a Model

In illustrating a systematic approach to graphic analysis, a model was chosen. A specific

musical piece serves as an example for the model, on the assumption that a method that works for such an example will have wider applicability.

The example for the model was chosen with the following criteria: (1) that the composition be complete, but short and uncomplicated, (2) for purpose of comparison, the composition should be one that Schenker also analyzed (although this should not be construed as an attempt to duplicate his graphs or methods), and (3) the music should clearly illustrate most of the general principles important to the method. The example chosen is J.S. Bach's C major *Prelude* from book I of *The Well-Tempered Clavier*. Schenker's analysis is found in a widely available collection, *Five Graphic Music Analyses* (Dover:1969). The algorithm consists of a series of concisely described steps for constructing the graphs, followed by a more detailed explanation of each, specifically referring to examples. As such, it is not a comprehensive model since not all aspects of domain determination, chromaticism, etc., occur in our example.

The Algorithm: Foreground

Foreground Verticalization, step 1 (Ex 1): Determine the key and prevailing harmonic rhythm from the score. Construct a graph notating all harmonic tones (chord tones) as verticalized half notes or white notes, and all non-harmonic tones as black-notes. Barlines and bar numbers are used at regular intervals for the purpose of location. Chord arpeggiations within each unit of the harmonic rhythm are verticalized.

A methodology for determining key and harmonic rhythm is foregone here since these methods are well known. In our example, the harmonic rhythm is one chord per measure, as shown in Ex 1. Only every fourth barline is retained, which helps to provide a framework for ordinary phrase structure. The actual phrase structure does not have to coincide thereof. The chord structure, as in this example, is tertian. For the sake of consistency, the normalized *maximum* number of pitch-classes for a chord is assumed to be four, *i.e.*, the number of pitch-classes for 7th-chords. There are almost twice as many 7th-chords (22) in the *Prelude* as there are triadic harmonies (12). The tetrachord of the 7th, then, is used as the basis of verticalized chord-tones in the foreground, *i.e.*, each white-note used in graphing must be part of a tertian structure of three or four pitch-classes and is shown vertically as such. All notes other than repetitions within a unit of the harmonic rhythm are included in Ex 1.

Foreground Chord Identity, step 2 (Ex 1): Label all harmonies with composite chord symbols (Roman numbers with figured bass), staying within the home key whenever possible.

Again the methodology is common knowledge and is therefore excluded here. In the *Prelude* these harmonies consist entirely of triads and 7th chords. Secondary-dominants and leading-tone chords are shown with a V/ or vii/ followed by the chord that each one "tonicizes". All chord-tones are drawn as half notes. Stems may be used to clarify voicings. In Ex 1 the bass clef stem includes tenor and bass voices, and the treble stem encompasses first and second soprano and alto voices. Non-chord tones are shown as black-notes. Notes are placed on the graph to show their relative temporal positions.

Foreground Domains (Ex 2): Determine and notate the foreground *harmonic-domains* with Roman numerals.

This crucial step involves the concept of the harmonic-domain. The music is divided into segments, each representing a harmonic function, chord, or set, that is "prolonged" by repetition or other means of emphasis in that segment; each of these prolongations is here called a harmonic-domain or simply, domain. Most tonal music is based on the harmonic-domains I, V, ii, or IV, each having a maximum of three pitch-classes; therefore, these are surveyed first for domain mapping. Each level of graphical analysis will have its harmonic-domains. The first task in this step is to locate and identify the foreground domains. The domains are shown as Roman numerals at the bottom of Ex 2, below the brackets, each indicating the prevailing harmony for a segment of the music. The following methodology is provided for domain mapping.

Two important harmonic-domains can be easily located since they form the final cadence. The final chord is identified as the final harmonic-domain. It is normally tonic. It is tested for prolongation by searching for repetitions or cyclic progressions at the close, moving backwards from the final cadence. The prelude contains no such repetition or cyclic progression. Therefore, the final domain is tonic, mapped as the last chord.

The harmony that precedes the final tonic, or its prolongation, is part of the cadence, normally the dominant, and it, too, must be considered an important harmonic-domain. The penultimate chord matches the dominant and is not prolonged. It is shown as V in Ex 2.

The initial harmonic-domain of a tonal work is ordinarily tonic, due to the ordinary establishment of key. Confirmation comes from a map of the initial chord progression, at least through the first phrase, consisting of a series of four chords or less. These are compared with some standard, cadential-formula progressions that are often used for establishing a key, such as, IV V I, ii V I, or I V I. If the key is not confirmed, a search may be performed for a longer progression or for chromatic notes which are then tested for probable alternative keys, *e.g.*, dominant and related keys. In the *Prelude*, the initial phrase contains the chord progression I ii⁴₂ V⁶₅ I, which cycles around the tonic in the first four measures. This contains one of the readily recognized, traditional cadential formula used to establish a key, and therefore confirms tonic as the initial domain. Three important domains have thereby been identified, the initial one and the final two.

Other harmonic-domains are located and identified by an examination of context. First, all chromatic notes are located. These are usually found contained in transient dominant and leading-tone 7th chords¹. A domain may be initiated by chromatically raised notes (sharps or natural accidentals) that function as leading-tones moving up a diatonic semitone to a new domain root, *e.g.*, Ex 2, measure 6 (F# leads to G, or V, the dominant), measure 12 (C# leads to D, or ii, the supertonic), and measure 22 (F# leads to G, or V). These domains are sometimes called "transient tonics" or, sometimes, "modulations", *i.e.*, harmonies that are emphasized by means of their leading-tones, which act as pointers. Arrows in the graph indicate these motions. Each occurs at the beginning of a new domain, shown below the brackets at the bottom of the graph with Roman numbers. The E flat in measure 28 does not meet this criteria since it moves up chromatically, and it is a flat.

Domain changes may also be initiated or confirmed by chromatically lowered notes (flats or naturals) in downward moving lines (diatonic step motion) that function as 7ths of secondary dominant or secondary leading-tone chords that are followed by their respective tonics. Diminished-

7th chords normally emphasize a root that is a major 3rd above the lowered note. Secondary dominant-7th chords emphasize roots that are a 4th below the chromatically lowered note. In Ex 2 the chromatic notes are shown with arrows in measure 12 (B-flat in the c# diminished-7th chord), measure 14 (A-flat in the b diminished-7th chord), measure 20 (B-flat in the C7 chord), measures 22-23 (E-flat in the f# diminished-7th), and measure 32 (B-flat in the C7 chord). There is also an A-flat in the b diminished-7th in measure 23 that presents a somewhat exceptional case. This chord is not followed by its normal resolution, C, but moves instead to G, a minor-2nd below the A-flat. This, along with the preceding f-sharp diminished-7th affirms the dominant domain.

The chromatic alterations mentioned are sufficient to establish all the domains in the graph of Ex 2. However, this criterion may not always serve as sufficient. Therefore, additional contexts are proposed as supplemental conditions for domains. Extended arpeggiation or repetition of a single chord can alone establish a domain, as can a statistical analysis of a chord that is found to dominate a segment of music. A static or pedal bass underlines the root of a domain, *e.g.*, measures 24-31 (G) and measures 32-35 (C). Rhythm can play an important role in the establishment of harmonic-domains, as the arbiter of emphasis; *e.g.*, strong and weak beats and cadential patterns. In the C major *Prelude*, the internal rhythm of measures does not play much of a part, due to the even flow of sixteenth notes. However, cadential patterns occur in measures 4, 11, 19, 24, and 35.

In addition to these, there are contexts that may establish domains in the absence of the former criteria. Normally they serve to corroborate domains; *e.g.*, the ends of sequences of root progressions up by 4th or down by 5th, near the chromatic notes. In Ex 2, the arrows between the Roman numerals show these movements. They mark domain boundaries, the commencement or termination of domains; *e.g.*, measures 4, 6, 11, 13, 14, 19, 21, 28-30, 33, and 34-35. A turning point, or change of direction, in the bass line leading to a goal-tone, can also help to confirm boundaries between harmonic-domains; *e.g.*, measures 4, 11, 19, 24, and 32. These are indicated with slurs below the bass line in Ex 2.

The Foreground

Foreground schematic (Ex 3a). Construct a foreground schematic. Prepare repeating notes for elimination by filling them in as black-notes.

A foreground schematic can be constructed from the information in Ex 2. Compare Ex 2 with Ex 3a. Note that the difference in Ex 3a consists in the addition of ties and filling in some notes, becoming black-notes, or secondary notes. The white and black-notes combinations with ties in Ex 3a indicate note repetitions. Each repetition consists of a single primary note and a secondary note (or notes), and these repetitions do not have to occur within the same voice part. The secondary note is filled in as a black-note. In most cases, the secondary note is one repeated in the next chord after its initial occurrence. However, if the initial occurrence of a repeated note anticipates a note in a domain chord (a domain chord is one that is identical to the domain), the first occurrence is the secondary note and is filled in, *e.g.*, the alto G of measure 3 anticipates the tonic chord in measure 4, a domain chord, and the soprano and tenor Ds in measure 6 anticipate the D of the following domain chord, V. Each white-note connected with a tie in Ex 3a marks the position of the primary note of a repetition. Each of these white-notes is to be retained in the foreground graph (Ex 3b), and each sufficiently represents all the immediate repetitions of that note within the domain. All the black notes in Ex 3a are

eliminated in Ex 3b.

In case of more than one repetition of a note, all repetitions after the initial occurrence are filled in (unless the note anticipates a domain chord), *e.g.*, the final repeating Cs in the bass, and the bass Gs of measures 24-31. However, if the repetition continues for more than four chords, the last note of the repetition is also kept as a white note, *e.g.*, in the repeating bass Gs from measure 24-31, the last note is white for this reason. This is to maintain the presence of the bass line where it may otherwise seem to drop out over a long pedal point.

Foreground (Ex 3b): Construct a *foreground* graph from the schematic, where only notes that are part of a domain chord are drawn as white-notes. All others are drawn as black-notes. The white-notes are then beamed together if they form arpeggiations of a single domain, using broken beams if the notes repeat in the same domain. Different beams are used to separate voices, but voices moving in parallel may be connected to the same beam. Black-notes must all be part of a line, moving stepwise, and these lines are shown by connecting the black-note stems to the beams of their respective domain notes, *i.e.*, the white-notes, or by using slurs if beaming is not sufficient to separate lines. Slurs are normally used to show step connection of lines moving from one domain to the next. If any black-note is not accounted for as part of a step (linear) motion, it is deleted. 7th and 9th displacements are sought and marked for white-notes that otherwise have no step connections.

Immediate bass motions by 4ths and 5ths within a domain are special movements that are shown with slurs. Each of these motions has an ancillary note and a principal note. The *ancillary* note is the lower note of a 4th movement, and it is the upper note of a 5th movement. The *ancillary* is shown as a separate note (unbeamed) with a stem and a flag.

Some of the white-notes in Ex 3a change to black-notes in Ex 3b, whereas, the black-notes of Ex 3a are omitted in Ex 3b.

The foreground domains are shown below the staff in Ex 3b. Any notes outside of these domains appear black in this graph. White-notes are stemmed and, in most cases, beamed together, each beam indicating a prolongation of a harmonic-domain. Notes can only be beamed together if they are within the same domain. Two types of beams are used: solid and broken beams. The broken beam is used to indicate a repetition of notes within the domain, while solid beams indicate an arpeggiation of the notes of a domain chord. In Ex 3b, domains that are not unravelling chords are shown as white-notes with no beam, *e.g.*, the V in measure 34 and the final tonic.

All black-notes in the foreground should be accounted for as step movements to and from white-notes. Otherwise, they are deleted. (None are deleted in our example). All are stemmed black-notes connected to beams unless there is some need to represent non-harmonic notes to fill in lines. Slurs are used to show stepwise motion connecting the lines between domain chords. They may also be used to show lines within a domain, especially when there are splits or convergences of lines to or from notes, *e.g.*, the bass F# and A-flat of measures 23-24 converge on a G in measure 24. Ex 3b also shows three "octave" displacements, which are actually couplings of a 7th and 9th, occurring in measures 33-35. Octave, 7th, and 9th displacements are shown with lines or slurs pointing up or down, in the direction of the displacement, along with an indication of the interval and direction of displacement on the diagrams. These are the equivalent of Schenker's *Koppelung*, a coupling or

register change. The 9ths are from A to B and from C to D, and a 7th occurs from F to E (tenor). These will have some significance for the middleground structure.

Measures 33-34 show a clear octave displacement. The B4 in the treble shows no stepwise approach, and the tenor line from the G3 in measure 29 that moves to A in 33 also has no stepwise exit. However, if the two are connected by a 9th displacement, the line shows a continuity that would otherwise not exist. A parallel movement connects the alto C4 (middle-C) in measure 33 to D5 in measure 34, corroborated by the same middle-C motion to D4 in measure 34. Likewise, the F3 in 33 can be approached from the same tenor G as a split or *divisi* from a unison, but where does it lead? Its expected continuation would be downward stepwise to E, which is corroborated by the alto F in 33 that moves down by step to E in the final chord. Thus, the low F of 33 is displaced a 7th upward to its resolution, E.

Notes that are repeated as pedals or are sustained within a single domain are shown with a single broken beam connecting the locations of the starting and ending notes; *e.g.*, the repeated G in the bass in measures 24-31.

White notes in the bass voice are examined for 4th and 5th movements *within a single domain*. Such motions are not beamed together in the foreground. Each of these motions has a principal and an ancillary note. If it is an upward leap of a 4th or a downward 5th, the principal note is the second note of the leap, and if it is an upward 4th or downward a 5th, the principal note is the first note. The ancillary note of each 4th or 5th leap is notated as a white eighth note with a separate flag and a slur connecting it to the principal note. Ex 3b shows two such leaps of a 4th in the bass, one in measures 10-11, and the other in measures 18-19.

The graph preserves the rhythmic structure through the use of the spatial notation of time. The physical layout of the graph represents the full length of the music; *i.e.*, there is virtually no horizontal compression. Each barline, except the first and last, in the foreground graph represents four in the original meter, and these are shown as equal in actual length on the page. The notes are graphically placed to reflect their virtual temporal positions as governed by the harmonic rhythm within each four measure frame, *e.g.*, Ex 3b through Ex 5. This makes the graph analogous to the convention of the "proportional" notation of rhythm found in many contemporary scores and makes it possible to represent an aspect of large scale harmonic rhythm in the analysis.

All notes of the original score, excepting their immediate repetitions, are accounted for in the foreground graph. Schenker did not normally do this, but it is important here in order to keep a consistent, systematic procedure. Playing/hearing the foreground with its proper rhythmic representation shows how closely it resembles the sound of the original music and yet distills the voice-leading, the harmonic structure, and the harmonic rhythm.

Middleground

Two primary functions of the middleground are: 1. to show a simplified tonal structure and voice-leading, and 2. to prioritize the structural elements and eliminate lesser details of the foreground to reveal a more fundamental or remote structure.

Middleground schematic A, step 1 (Ex 4a): Eliminate the black-notes of the foreground. This schematic should display only notes of the foreground domains. Delete all slurs, and consolidate beams and stems.

The first step in the construction of the middleground is to eliminate the black-notes and slurs of the foreground graph. Ex 4a shows the result of this. Compare Ex 3b with Ex 4a. Some notes from Ex 3b are combined on one stem and beam in Ex 4a to simplify the appearance of the graph.

Middleground schematic A, step 2 (Ex 4a): Extract and label the middleground domains as simplifications of the foreground domains from the foreground graph.

A deeper level of harmonic-domains are determined from the foreground through a simplification of the existing foreground domains and the bass line. The primary harmonies to locate are the same as before, I, V, ii or IV. Once again, we first examine the final cadence. The last tonic is the final domain. It is not prolonged by repetition or cyclic progression, but is preceded by a dominant. This dominant shows a higher level of prolongation through the cyclic progression from measures 22 to 34; , the progression IV V IV V of the foreground reduces to the dominant domain of the middleground.

The first series of three or four foreground harmonic-domains are examined for cyclic motions around I and V, or for common progressions that emphasize these tonal areas (just as in determining the foreground domains), normally reinforced by the rhythm or the contour of the bass line. The domain, tonic or dominant, is identified from this. The first such group found in Ex 4a is I V ii6 I, a cycle around tonic. This is reinforced and outlined by a bass line that descends rather consistently as a C Major scale from C4 (middle C) to C3 before breaking at measure 20 to the low F2. Thus, the first middleground domain is tonic, from the beginning to measure 20.

Finally, if any chords are left from the foreground domains, they are examined for movements that emphasize ii or IV. There are none in our example. Summarily, the middleground domain progression is I V I as shown in Ex 4a.

Middleground schematic B (Ex 4b): Eliminate all repeated notes.

Repeated white-notes that were shown with broken beams in Ex 4a are reduced to their initial occurrence in Ex 4b, *e.g.*, the repeated C major chord at the beginning. However, if another voice part is difficult to separate on the diagram, obscuring the voice-leading, the broken beam is retained. This occurs in the bass G in measure 31, which would, if reduced to the initial G in measure 24, seem to move to the tenor D in measure 26 and obscure the prolongation of the dominant.

Middleground schematic C (Ex 4C): Using the middleground schematic B, construct middleground schematic C, where only middleground domain notes are white, and the remaining notes are black.

The notes of the simpler and broader middleground domains become the only white-notes in

middleground schematic C. All other notes are black, which will be shown in the context of linear, step motions to and from white-notes. This change can be easily made by filling in the non-domain notes of the previous schematic.

Middleground schematic D, step 1, (Ex 4d). Locate all black-note step progressions to and from domain notes. All white-notes within each domain are beamed together as step motions from one white-note to another, using the black-notes as steps to fill in the arpeggiations. Stems are added that connect the black-notes to a white-note beam that reflects its voice leading context. A separate beam is used for each voice whenever possible, but some voices may be combined on one beam if they move in parallel or to avoid clutter. The same rules apply to the use of slurs as in the Foreground, showing the voice leading between domains. Only the bass voice may move by leap, and only whenever no step motion exists between leaps. Reinstate any notes from the foreground that are necessary to complete step motion within each voice. 7th and 9th displacements are sought for white-notes that otherwise show no stepwise voice-leading.

Beaming the notes of the middleground domains traces their linear, step contexts. However, the notes from middleground schematic C are not sufficient to show every linear context since some white note arpeggiations remain without step motion. This is sometimes permissible in the bass, but not so in other voices. After all the possible step motions are marked, there remain some white-note leaps for which there appear to be no stepwise connection, *e.g.*, in measure 24, the soprano D appears to leap to a G at measure 29 (see schematic C). However, by backtracking to the foreground graph we can see that there is step movement up to the G. It is sufficient to reinstate only the diatonic notes E and F into the middleground graph. These are circled in Ex 4d. A few other circled notes appear in Ex 4b. All these are notes lifted from the foreground and reinstated in the middleground to show the step motion of the voices. The **only** notes that may be reinstated from the foreground are passing-tone motions that are missing but necessary to show a stepwise voice-leading between white-notes in the middleground. By reinstating the A3 and D3 circled in the bass line we can see that the bass moves from C4 down a complete major scale to C3 in measure 19. There are a total of six of these reinstated notes. Some notes in the inner voices may change clef in order to more clearly show their linear paths.

Three voice-leading displacements are shown in Ex 4d that we have previously explained in the foreground graph. These are the 7th and 9th displacements near the close. Two others appear on Ex 4d. The A in measure 33, shown with the upward 9th motion, serves as a link between the soprano G in measure 29 and the B of measure 34, through a downward 7th displacement.

The soprano C in measure 15 appears to end the soprano line in the tonic domain, but the first soprano note in the dominant domain is D, serving as a stepwise link by 7th displacement. The two 7th couplings are reinforced by octave duplication of these lines. The 7th coupling from G in measure 29 to A in measure 33 is corroborated by the same motion in the tenor without any displacement. The F to E 7th coupling in the tenor at the end is corroborated by the alto having the same motion without displacement.

Middleground schematic D, step 2 (Ex 4d): Locate and mark doubled lines. Designate repeating notes within each voice part with a tie. Tag notes that have no step connection or are otherwise duplications by enclosing each in parentheses.

A comparison of voices shows that some are doubling the lines of another voice in parallel octaves. These doubled lines are shown with note names connected by dashes above and below the staff; *e.g.*, at the beginning, the soprano line, E-D-C, is doubled in the tenor voice. The tenor line, B-A-G is doubled in the second-soprano. In measure 21-24 the tenor and bass move in octaves from F to G. In measures 29-35 the alto moves G-F-E. This is duplicated in the tenor from measure 24, with a 7th displaced E at the end.

Repeating white or black-notes within a domain line are located and shown with ties. Notes without step connections are shown in with an X (the soprano G in measure 7 leads nowhere, so it and the F that leads to it are marked). Unnecessary duplications are similarly marked (the alto D in measure 7 no longer serves a separate function from its unison in the tenor). These will be eliminated in the next step.

Middleground (Ex 4e): Construct the middleground graph from the information in the last schematic, beaming the arpeggiated notes of each middleground domain together, voice by voice, and displaying the black-notes as step motions that connect the white-note arpeggiations. Use a separate beam for each voice part when possible. Reduce doubled lines to a single voice, eliminating the doubled part in this order: TAS2 (tenor, alto, second-soprano); *i.e.*, a tenor part is the first to be deleted when doubled. Never eliminate bass lines. Reduce repeating notes (tied notes in Ex 4d) to their initial occurrence. Delete any notes that are not part of a stepwise-moving line, excepting the bass, and delete ancillary bass notes.

The reduced voicing is transferred to the middleground graph, Ex 4e. There are still doubled notes in the graph; each doubled note shows voice-leading that is unique. For example, the C in the second-soprano at the beginning is retained even though it is doubled in the bass; it shows a different part (the same line imitated). The doubled B in the penultimate measure is kept to show its unique approach in the separate voices, one from above, the other as an 9th displacement from below. The high G in the soprano in measure 7 is eliminated because it leads nowhere stepwise on this level.

The bass voice is the only voice that may have white-note motion that is not stepwise, and these are normally 4th or 5th leaps, such as V I or I V progressions; *e.g.*, the last bass beam. The last two domain bass notes forming the broad cadential movement dominant to tonic, or G2 to C3, have been beamed together in Ex 4e. This is normal in the middleground and background but is the only case that different domain notes may be beamed together in the middleground.

Once the middleground is constructed, we can observe some interesting features that were not previously obvious. The bass line descends an octave from middle-C in measures 1-20, then moves to the dominant (measure 24), where it remains for several measures, and moves to tonic in the final bar. The soprano starts on a high E and descends to C in measure 15. A second soprano beam in the dominant domain shows the soprano continuing from D up to G in measures 23-29 and further through 7th and 9th displacements to A, B, and finally C; thus, the entire soprano line is a falling 3rd from E to C followed by an ascending C major scale with octave displacements. Splitting from this line, in measure 29, is an alto line, G-F-E, shown with the alto beam. This final E4 is joined from below by an ascending tenor line from measure 15 that moves upward, G-A-B-C-D-E. However, the complete story of the tenor comes from the beginning bass and second-soprano line descending from tonic to dominant, coupled through a 7th displacement to A3 in measure 21 in the dominant domain (also converging on the A from the G tenor in the bass clef in measure 15). Thus, the complete tenor line descends from tonic to dominant and then ascends back to tonic and up to E4.

The middleground projects the rhythmic structure of a more fundamental voice-leading and

harmonic motion. Again, playing/hearing it reveals significant structures that are not readily apparent.

The Background

Background schematic (Ex 5a): Delete all black-notes of the middleground graph. Verticalize the arpeggiation of each middleground domain. Locate and mark all voice doublings. Reduce the entire graph to a tonic domain, i.e, as white notes.

First, all the black-notes of the middleground are deleted. The white-notes under each beam of the middleground are verticalized, as in Ex 5a. Tonic is the only domain in the background. The new black-notes of the background are notes outside of the tonic domain. Notes of the tonic are white and beamed together, voice by voice. Location and marking of duplicate voicings follows the same criteria as in the middleground procedure. The remaining notes form the basic outline of the background, or *Ursatz*.

Background or *Ursatz* (Ex 5b): Eliminate all voice part duplications in the order TAS. Construct the background graph, or *Ursatz*, by beaming the notes of the unravelling tonic voices together on each staff and showing the linear movements of the upper voices.

We can see from the domain structure of the middleground, I V I, that this can be reduced to simply I, or tonic; however, the dominant symbol is also retained here, honoring the common practice for the background. However, only members of the tonic chord are now white-notes.

The fundamental soprano line, or *Urlinie*, is seen in Ex 5c as a 3-2-1 scalar motion and can be labeled as such with accent marks if desired. The notes of the unravelling tonic are beamed in the background, with the black-notes shown as parts of lines, as before. Only one black-note, the F4 from the alto in measure 30, is reinstated from the middleground, in order to show the alto line moving from G to E through the passing-note F.

A Comparison with Schenker's Analysis

In Schenker's own analysis of the C major *Prelude*, shown in Ex 6, his most detailed graph, the foreground (*Vdg.*), the bottom level, is comparable to our Ex 2 and Ex 3b combined. His reduction proceeds more rapidly from one level to the next, and some of the symbols are different, but the essential meanings of the graphs are comparable. The initial step of verticalization is immediately apparent in Schenker's first (lowest) graph. However, there are a few details in his graph that are not shown in our model. For instances, the large scale parallel-10th (*Oberdezimen*) movement between soprano and bass from the beginning to measure 19, and the large scale 4th and 5th motions in the bass (*Quartzug* and *Quintzug*). Schenker also shows the *Urlinie* (fundamental soprano line) in his foreground graph. This is reserved for our final stage, Ex 5c, the *Ursatz*. Generally, Schenker incorporates some of the large scale analysis on his lowest level, whereas our algorithmic method has built in controls that allow these large scale structures only in the last stages of the analysis.

On the other hand, there are many aspects of voice-leading and tonal hierarchy shown in the algorithmic graphs that are not in Schenker's. This includes most of what is in our foreground and middleground graphs; *e.g.*, the contextual beams show many lines that are completely absent from Schenker's graphs. The hierarchical harmonic-domains are also absent. The algorithmic graphs clearly show much more detail.

In some cases, Schenker's observations are shown with a different symbol by the algorithm; *e.g.*, many of the contextual slurs he used in the foreground are found with beams in Ex 3b and Ex 4c. The Arabic numbers below the staff in Schenker's graphs, which outline the voice-leading, are shown graphically with beams or slurs in the algorithmic foreground.

There are some basic disagreements, too. First, some of Schenker's notes are in error. The alto E in measure 15 should be a C, and the tenor A in measure 30 should be a G. More serious, however, is the deletion of the leading-tone, B, in measure 23. From a published letter concerning this measure, we know that this is not an oversight, but Schenker's only explanation for its obliteration is that Bach merely "feigns" the outline of a 3rd.ⁱⁱ This is the only note that Schenker omitted from the score; that is, until measures 33-34, where the omissions are understood as the beginning of note eliminations in this cadenza-like passage. It is difficult to justify his intent. The leading tone is not superfluous to the structure, and surely Bach did not put it there merely as any note to fill in the finger work. Schenker's omission of the leading-tone is unique in measure 23, and leads him into an erroneous interpretation of its context. He labels the chord there as a II^6_5 (it certainly is a II if the B is ignored). What is more surprising is that Schenker actually deletes the leading-tone and keeps the result as a significant harmony in his middleground. However, with the B present, the chord is a leading-tone 7th, not a II^6_5 (curiously, he seems to show its position differently in his *Ursatz*). The B cannot be ignored, and its presence considerably weakens the credibility of II as a unique middleground domain. This chord is simply a continuation of the dominant-preparation begun by the IV preceding it.

Schenker gives a great deal of attention to the chromatic motion in measures 22-23, and even includes an exclamatory note on the bass there. If the leading-tone were not deleted in his analysis, it seems certain that the result would have been different, and the II would then have to be removed as a significant middleground harmony.

Another significant difference in Schenker's analysis is the position of the final tonic (domain). He considers the tonic to be prolonged from measures 32-35, presumably because of the bass pedal in these measures (curiously, he shows its position differently in his *Ursatz*). The aspect of a prolonged bass is considered in the algorithm, but aside from the bass C, there is no other corroboration for the tonic until measure 35. In fact, a close examination shows measures 32-33 to be a IV^6_4 , a common cadential six-four, until measure 34 where the dominant occurs, immediately followed by the final tonic.

In a level-by-level comparison, Schenker's *Ursatz* is comparable in meaning, if not in appearance, to the results achieved by the algorithm. His foreground and middleground are much less detailed but what is there is contained in the middleground graph of Ex 4c, notwithstanding the differences just mentioned. The foreground graph, Ex 3b, has no direct counterpart in Schenker's analysis, but instead, the foreground schematic, Ex 2, is a parallel to Schenker's foreground, with the exception of Schenker's errors in copy, mentioned above. Additionally, his contextual slurs are mixtures of the separate contexts defined for slurs and beams incorporated in the foreground graph of Ex 3b.

In conclusion, the algorithm designed here gives results comparable, but not identical, to those obtained by Schenker, yet yields more structural detail. More important is the application of a consistent procedure that may be followed for other analyses. This procedure has been designed to

illuminate the structural aspects of the harmony and voice-leading, but also contains aspects of the formal and rhythmic structure.

NOTES

Schenkerian Graphs: Bach's C Major Prelude, WTC I

The graphs on the following pages comprise an example analysis using the foregoing algorithm. These are graphs of J.S. Bach's *Prelude*, No. 1 in C Major from the *Well-Tempered Clavier, Book I*, which is contained in the *Anthology of Musical Structures*.

Appendix B. Symbols for Schenkerian Graphs



Half Notes (or other “white notes”) indicate notes that are a part of an important structural harmony, or domain.



Quarter Notes indicate motion (normally stepwise) to and from domain notes.



Unstemmed black notes may be used to show most immediate level of voice leading when other notes are insufficient.



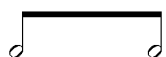
Different types of notes from longest to shortest indicate differing structural levels of voice leading, from remote to immediate.



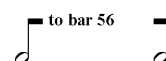
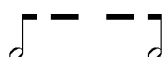
Notes with separate flags indicate immediate auxiliary notes, such as incomplete neighbors and bass leaps of 4ths or 5ths. They should be accompanied by a slur or arrow to indicate the primary note of function.



Parenthesized notes represent those that are implied but not actually present, or they represent notes marked for deletion on the next structural level.



Beamed notes connect notes in one domain harmony. They are the notes that are part of this harmony; i.e., harmonic tones, while intervening black notes connected to the same beam show the stepwise voice leading between domain notes. A solid beam indicates a chord arpeggiation, while a broken beam represents note repetitions. Other authors use beams to show large scale connections in general.



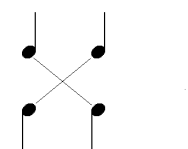
A broken beam with “to” indicator represents a long term connection.



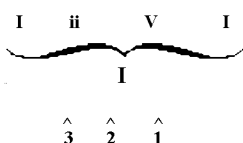
A slur shows the voice leading connections between one domain and the next domain, although they are most often used by other authors to show voice leading in general. The broken slur is used to show voice leading when notes repeat.



Lines, arrows, and ess-shaped slurs represent changes of register, octave displacements, or displacements of a line by a 7th, 8ve, 9th, etc. These should be accompanied by a number that indicates the interval displacement.



Voice exchange is shown with crossing lines between the voices involved. Interruptions are represented by two parallel, vertical lines.



Dominance of a single harmony over a large scale is represented by domains, shown with brackets or braces. Remote symbols are largest.

A structural soprano line may be shown with carreted numbers. Uncarreted numbers may be used for more immediate levels in any voice.



Unfolding; a single part splits into two voices



Fastest note values indicate most immediate level of voice leading. They may also be used for immediate chord arpeggiations.

Other Recommended Symbols



Circled notes represent reinstated notes from a previous level of structure.



X-notes may be used for notes to be deleted. This is recommended instead of using parenthesized notes (see above) to keep separate from implied notes.



Octave doublings may be shown this way, with the voice abbreviations in parentheses. This also marks the doubled inner voice for deletion.

Useful Abbreviations Found on Schenkerian Graphs

Arp	arpeggiation
CS	consonant skip (leap, arpeggiation)
D or Div	divider, usually a V chord, that marks sections.
DF	chord of double function; i.e., one that exhibits both immediate and remote voice leading functions.
IN	incomplete neighbor
NC	passing or neighbor chord
Kopp	coupling (German: <i>Koppelung</i>); i.e., showing octave transfer or register change
LN	lower neighbor
N	neighbor
P	passing
RT	register transfer
UN	upper neighbor
6 - 5 - 6	Arabic numbers may be used for scale degrees or for intervals.
10 - 10 - 10	
V4	Roman numbers and figured bass are used for chords.

1. Of course, it is recognized that chromatic notes may have other functions; e.g., color, auxiliary notes, etc. These are not accounted for in our algorithm since these functions do not occur in the example.

2. Drabkin, William. "A Lesson in Analysis from Heinrich Schenker: The C Major Prelude from Bach's Well Tempered Clavier, Book I", *Music Analysis*, Vol. 4, No. 3, 1985, pp. 255-257.